Task:

A catapult launches a test rocket vertically upward from a well, giving the rocket an initial speed of 80.2 m/s at ground level. The engines then fire, and the rocket accelerates upward at 3.80 m/s2 until it reaches an altitude of 1190 m. At that point its engines fail, and the rocket goes into free fall, with an acceleration of −9.80 m/s2. (You will need to consider the motion while the engine is operating and the free-fall motion separately.)

Solution:

$$s_0 = 0, v_0 = 80.2\frac{m}{s}, a = 3.8\frac{m}{s^2}, s = s_0 + v_0 t + \frac{at^2}{2}$$

$$s_1 = 0 + 80.2\frac{m}{s}\cdot t + \frac{3.8\frac{m}{s^2}\cdot t^2}{2}, 1190\,m = 0 + 80.2\frac{m}{s}\cdot t + \frac{3.8\frac{m}{s^2}\cdot t^2}{2}$$

$t = 11.6323\ s$ – the time after launch with working engines

$$v_1(t) = v_{2_0} = v_0 + at = 80.2\frac{m}{s} + 3.8\frac{m}{s^2}\cdot 11.6323\ s = 124.4027\frac{m}{s}$$

$$s_2 = 1190\ m + 124.4027\frac{m}{s}\cdot t - \frac{9.8\frac{m}{s^2}\cdot t^2}{2}$$

$$0 = 1190\ m + 124.4027\frac{m}{s}\cdot t - \frac{9.8\frac{m}{s^2}\cdot t^2}{2}$$

$t = 32.7939\ s$ – the time after engines fail till the crash

$t = 44.4262\ s$ –total time of the motion

Mathematical model of the motion: