

QUESTION:

In a women's 100-m race, accelerating uniformly, Laura takes 1.82 s and Healan 3.07 s to attain their maximum speeds, which they each maintain for the rest of the race. They cross the finish line simultaneously, both setting a world record of 10.4 s.

I guess, that the question is :

- a) which sprinter is ahead at the 6.15-s mark, and by how much?
- b) what is the maximum distance by which Healan is behind Laura?
- c) At what time does that occur?

SOLUTION:

Let's denote Laura's acceleration, speed and distance covered as a_L , v_L and d_L accordingly.

a_H , v_H and d_H are Healan's acceleration, speed and distance covered.

Also $s = 100\text{m}$, $t_L = 1.82\text{ s}$, $t_H = 3.07$, $t_R = 10.4\text{ s}$

While accelerating uniformly Laura runs

$$d_{L,a} = \frac{a_L t_L^2}{2}$$

and she reaches maximum speed

$$v_L = a_L t_L$$

After that she maintain constant velocity and runs

$$d_{L,const} = v_L (t_R - t_L) = a_L t_L (t_R - t_L)$$

Whole distance traveled by Laura is

$$s = d_{L,a} + d_{L,const} = \frac{a_L t_L^2}{2} + a_L t_L (t_R - t_L)$$

Analogically, whole distance traveled by Healan is

$$s = \frac{a_H t_H^2}{2} + a_H t_H (t_R - t_H)$$

We can now find Laura's and Healan's accelerations:

$$\frac{a_L t_L^2}{2} + a_L t_L (t_R - t_L) = s$$

$$a_L \left(\frac{t_L^2 + 2t_L(t_R - t_L)}{2} \right) = s$$

$$a_L = \frac{2s}{t_L^2 + 2t_L(t_R - t_L)}$$

$$a_L = 5.79\text{ m/s}^2$$

Analogically

$$a_H = \frac{2s}{t_H^2 + 2t_H(t_R - t_H)}$$

$$a_H = 3.67\text{ m/s}^2$$

At $t_1 = 6.15\text{ s}$ mark Laura runs

$$d_L = \frac{a_L t_L^2}{2} + a_L t_L (t_1 - t_L)$$

$$d_L = 55.22\text{ m}$$

And Healan runs

$$d_H = \frac{a_H t_H^2}{2} + a_H t_H (t_1 - t_H)$$

$$d_H = 52.06 \text{ m}$$

So, at 6.15 s mark Laura is the first. The distance between them at this moment is $d_b = d_L - d_H = 3.16 \text{ m}$

At $t=3.07 \text{ s}$ Healan reaches her maximum speed and the distance between she and Laura begins to shorten. Hence, the maximum distance between them is at the period of time, when Laura reaches her maximum speed, and Healan accelerates:

$$d_{LH} = \frac{a_L t_L^2}{2} + a_L t_L (t - t_L) - \frac{a_H t^2}{2}$$

$$d_{LH} = -\frac{a_H}{2} \cdot t^2 - a_L t_L \cdot t - a_L t_L^2 + \frac{a_L t_L^2}{2}$$

d_{LH} is quadratic function and it has maximum at

$$t_{\max} = -\frac{a_L t_L}{-2 \frac{a_H}{2}} = \frac{a_L t_L}{a_H} = 2.87 \text{ s}$$

And maximum distance between Laura and Healan is

$$d_{LH,\max} = -\frac{a_H}{2} \cdot t_{\max}^2 - a_L t_L \cdot t_{\max} - a_L t_L^2 + \frac{a_L t_L^2}{2} = 5.52 \text{ m}$$

ANSWER:

- a) So, at 6.15 s mark Laura is the first. The distance between she and Healan is 3.16 m
- b) Maximum distance between Laura and Healan is $d_{LH,\max} = 5.52 \text{ m}$
- c) Maximum distance between Laura and Healan is at $t_{\max} = 2.87 \text{ s}$