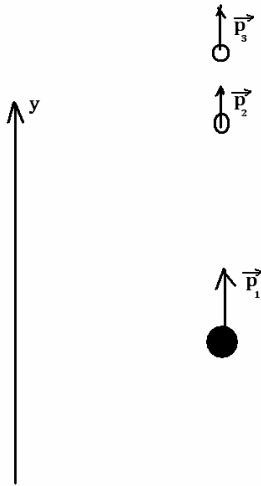


QUESTION:

A projectile of mass $m_1 = 50\text{kg}$ is shot vertically upwards with an initial velocity of $u_0 = 100\text{m/s}$. After $t_1 = 5\text{s}$, it explodes into two fragments, one of which having mass $m_2 = 20\text{kg}$ travels vertically up with a velocity of $u_2 = 150\text{m/s}$. What is the velocity u_3 of the other particle at that instant? Calculate the sum of momenta of the fragments $t_2 = 4\text{s}$ after explosion and show that it is equal to the momentum of the projectile at that instant if there were no explosion. (take $g = 9.8\text{m/s}^2$)

SOLUTION:

(Attention – the length of the momentum vectors on this sketch isn't proportional to their modulus)

Let's find the velocity of the projectile in the moment of explosion:

$$v_1 = v_0 - g \cdot t_1 = 100 - 9.8 \cdot 5 = 51 \text{ m/s}$$

According to the momentum conservation law:

$$\vec{p}_1 = \vec{p}_2 + \vec{p}_3$$

$$m_1 v_1 = m_2 v_2 + m_3 v_3 \quad (\text{projection on y axis})$$

Hence

$$v_3 = \frac{m_1 v_1 - m_2 v_2}{m_3} = \frac{50 \cdot 51 - 20 \cdot 150}{50 - 20} = -15 \text{ m/s}$$

(Here minus sign show, that velocity v_3 is directed downward, and its projection on y axis is negative. We can't predict the direction of v_3 velocity without calculation and hence we assume first that it was directed upward, but calculations show that that velocity v_3 is directed downward)

Let's find the velocities and momenta of the fragments after 4 s after explosion:

1st fragment :

$$v'_2 = v_2 - gt = 150 - 9.8 \cdot 4 = 110.8 \text{ m/s}$$

$$p'_2 = m_2 v'_2 = 2216 \text{ kg} \cdot \text{m/s}$$

2nd fragment

$$v'_3 = v_3 - gt = -15 - 9.8 \cdot 4 = -54.2 \text{ m/s}$$

$$p'_3 = m_3 v'_3 = -1626 \text{ kg} \cdot \text{m/s}$$

$$p'_2 + p'_3 = 590 \text{ kg} \cdot \text{m/s}$$

Let's find now the velocity and momentum of projectile after $4s+5s=9s$ from shot, assuming that there was no explosion:

$$v'_1 = v_0 - g \cdot (t_1 + t_2) = 100 - 9.8 = 11.8 \text{ m/s}$$

$$p'_1 = m_1 v'_1 = 590 \text{ kg} \cdot \text{m/s}$$

Hence

$$p' = p'_2 + p'_3$$

ANSWER

$$v_3 = -15 \text{ m/s}$$

$$p' = p'_2 + p'_3 = 590 \text{ kg} \cdot \text{m/s}$$