Suppose a spring-mass system has $k = 18Nm^{-1}$ and $m = 0.71kg$. The system is oscillating with an amplitude of $54mm$. (i) Determine the angular frequency of oscillation. (ii) Obtain an expression for the velocity v of the block as a function of displacement x and calculate v at $x = 34mm$. (iii) Obtain an expression for the mass's distance $|x|$ from the equilibrium position as a function of the velocity v and calculate $|x|$ when $v = 0.18ms^{-1}$. (iv) Calculate the energy of the spring-mass system.

Solution.

$$k = 18\frac{N}{m}, m = 0.71kg, x_{max} = 54mm = 0.054m, x = 34mm = 0.034m, v = 0.18\frac{m}{s};$$

(i) The angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}}.$$

$$\omega = \sqrt{\frac{18}{0.71}} = 5\left(\frac{rad}{s}\right).$$

(ii) The kinetic energy at the any moment:

$$E_k = \frac{mv^2}{2}.$$

The potential energy at the any moment:

$$E_k = \frac{kx^2}{2}.$$

The total amount of energy:

$$E = E_{kmax} = E_{pmax}.$$

$$E = E_{pmax} = \frac{kx_{max}^2}{2}.$$

The total amount of energy at the any moment:

$$E = E_k + E_p;$$

$$\frac{kx_{max}^2}{2} = \frac{mv^2}{2} + \frac{kx^2}{2}.$$

An expression for the velocity v of the block as a function of displacement x:

$$\frac{mv^2}{2} = \frac{kx_{max}^2}{2} - \frac{kx^2}{2};$$

$$v^2 = \frac{k}{m}(x_{max}^2 - x^2);$$

$$v = \sqrt{\frac{k}{m}(x_{max}^2 - x^2)}.$$

$$v = \sqrt{\frac{18}{0.71}(0.054^2 - 0.034^2)} = 0.21\left(\frac{m}{s}\right).$$

(iii) The total amount of energy at the any moment:

$$E = E_k + E_p;$$

$$\frac{kx_{max}^2}{2} = \frac{mv^2}{2} + \frac{kx^2}{2}.$$

An expression for the mass's distance $|x|$ from the equilibrium position as a function of the velocity v:

$$\frac{kx^2}{2} = \frac{kx_{max}^2}{2} - \frac{mv^2}{2};$$

$$x^2 = x_{max}^2 - \frac{m}{k}v^2;$$

$$x = \sqrt{x_{max}^2 - \frac{m}{k}v^2}.$$

$$x = \sqrt{0.054^2 - \frac{0.71}{18}0.18^2} = 0.04(m).$$

(iv) The energy of the spring-mass system:

$$E = E_{pmax} = \frac{kx_{max}^2}{2}.$$

$$E = \frac{18 \cdot 0.054^2}{2} = 0.026(J).$$

Answer:

(i) $\omega = 5\frac{rad}{s}$.

(ii) $v(x) = \sqrt{\frac{k}{m}(x_{max}^2 - x^2)}. v(0.034) = 0.21\frac{m}{s}$.

(iii) $x(v) = \sqrt{x_{max}^2 - \frac{m}{k}v^2}. x(0.18) = 0.04m$.

(iv) $E = 0.026J$.