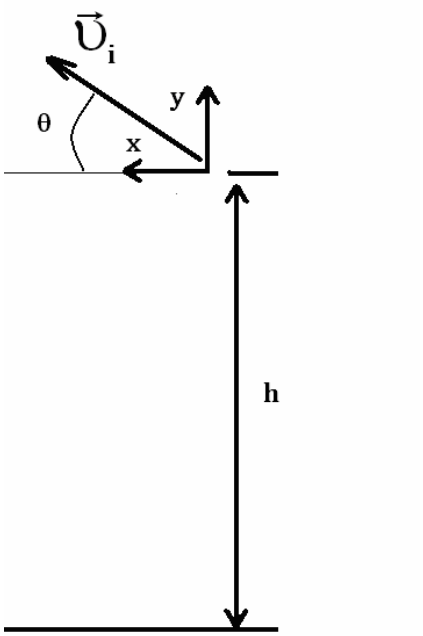
QUESTION:

A projectile is fired from the top of a cliff of height h above the ocean below. The projectile is fired at an angle θ above the horizontal and with an initial speed vi.(a) Find a symbolic expression in terms of the variables vi, g, and θ for the time at which the projectile reaches its maximum height.(b) Using the result of part (a), find an expression for the maximum height hmax above the ocean attained by the projectile in terms of h, vi, g, and θ

SOLUTION:



The equations of motion of a projectile in chosen origin are:

$$\begin{cases} x = \upsilon_i \cos\theta \cdot t \\ y = \upsilon_i \sin\theta \cdot t - \dfrac{g \cdot t^2}{2} \end{cases}$$

The velocity's projection

$$\begin{cases} \upsilon_x = \upsilon_0 \cos\theta \\ \upsilon_y = \upsilon_0 \sin\theta - g \cdot t \end{cases}$$

When a projectile reaches the maximum height, the y-projection of its velocity becomes zero, hence we can find the time t_m , it takes the projectile to reach the maximum height:

$$0 = \upsilon_0 \sin\theta - g \cdot t_m$$

$$t_m = \frac{\upsilon_0 \sin\theta}{g}$$

Hence, the maximum height (above the top of the cliff) is

$$y_m = \upsilon_0 \sin\theta \cdot \frac{\upsilon_0 \sin\theta}{g} - \frac{g}{2}\left(\frac{\upsilon_0 \sin\theta}{g}\right)^2 = \frac{\upsilon_0^2 (\sin\theta)^2}{2g}$$

the maximum height above the ocean is

$$h_{max} = h + y_m = h + \frac{\upsilon_0^2 (\sin\theta)^2}{2g}$$

ANSWER:

$$t_m = \frac{\upsilon_0 \sin\theta}{g}$$

$$h_{max} = h + \frac{\upsilon_0^2 (\sin\theta)^2}{2g}$$