

Task:

A 1.70 m wide, 1.40 m high, 1300 kg car hits a very slick patch of ice while going 20.0 m/s. Air resistance is not negligible. How long will it take until the car's speed drops to 16 m/s?

Solution:

For the car the drag force will be:

$$F_{drag} = -\frac{1}{4} \cdot width \cdot height \cdot v^2 \cdot \frac{kg}{m^3}$$

Newton's 2nd Law:

$$\sum_{i=1}^n F_i = \frac{dp}{dt} = M \frac{dv}{dt}$$

$$\sum_{i=1}^n F_i = F_{drag}$$

$$M \frac{dv}{dt} = -\frac{1}{4} \cdot width \cdot height \cdot v^2 \cdot \frac{kg}{m^3}$$

$$\frac{dv}{v^2} = -\frac{1}{4 \cdot M} \cdot width \cdot height \cdot \frac{kg}{m^3} \cdot dt$$

$$\int_{\frac{20 \frac{m}{s}}{16 \frac{m}{s}}} \frac{dv}{v^2} = -\int_0^t \frac{1}{4 \cdot M} \cdot width \cdot height \cdot \frac{kg}{m^3} \cdot dt$$

$$-\frac{1}{v} \Big|_{\frac{20 \frac{m}{s}}{16 \frac{m}{s}}} = -\frac{1}{4 \cdot M} \cdot width \cdot height \cdot t \cdot \frac{kg}{m^3} \Big|_0^t$$

$$-\frac{1}{16 \frac{m}{s}} + \frac{1}{20 \frac{m}{s}} = -\frac{1}{4 \cdot M} \cdot width \cdot height \cdot t \cdot \frac{kg}{m^3}$$

$$-\frac{1}{80 \frac{m}{s}} = -\frac{1}{4 \cdot 1300 \text{ kg}} \cdot 1.7 \text{ m} \cdot 1.4 \text{ m} \cdot t \cdot \frac{kg}{m^3}$$

$$t = \frac{\frac{1}{80 \frac{m}{s}}}{\frac{1}{4 \cdot 1300 \text{ kg}} \cdot 1.7 \text{ m} \cdot 1.4 \text{ m} \cdot \frac{kg}{m^3}} = \frac{4 \cdot 1300}{80 \cdot 1.7 \cdot 1.4} \text{ s} \approx 27.311 \text{ s}$$

Answer:

$$t \approx 27.311 \text{ s}$$