Task:

The period of the earth around the sun is 1 year and its distance is 150 million km from the sun. An asteroid in a circular orbit around the sun is at a distance 259 million km from the sun. What is the period of the asteroid's orbit? Answer in units of year. What is the orbital velocity of the asteroid?
Assume there are 365 days in one year. Answer is in units of m/s

Solution:

Consider the distance to be the distance between centers of objects.

$t_{earth} = 1\ year$

$l_{earth} = 2\pi R_{earth}$

From Newton's 2 law:

$$F = M_{earth}a = \frac{M_{earth}v_{earth}^2}{(R_{earth})}$$

Gravitational law:

$$F = \frac{GM_{earth}M_{sun}}{(R_{earth})^2}$$

$$\frac{GM_{earth}M_{sun}}{(R_{earth})^2} = \frac{M_{earth}v_{earth}^2}{(R_{earth})}$$

$$\frac{GM_{earth}M_{sun}}{(R_{earth})} = M_{earth}v_{earth}^2$$

$$v_{earth}^2 = \frac{GM_{sun}}{(R_{earth})}$$

$$M_{sun} = \frac{v_{earth}^2(R_{earth})}{G} = \frac{\frac{l_{earth}^2}{t_{earth}^2}(R_{earth})}{G} = \frac{4\pi^2 R_{earth}^3}{Gt_{earth}^2}$$

$$v_{earth} = \sqrt{\frac{GM_{sun}}{(R_{earth})}}$$

$$\frac{l_{earth}}{t_{earth}} = \sqrt{\frac{GM_{sun}}{(R_{earth})}}$$

$$v_{asteroid} = \frac{l_{asteroid}}{t_{asteroid}} = \sqrt{\frac{GM_{sun}}{(R_{asteroid})}} = \sqrt{\frac{4\pi^2 R_{earth}^2 G}{(R_{asteroid})Gt_{earth}^2}} = \sqrt{\frac{4\pi^2 R_{earth}^3}{(R_{asteroid})t_{earth}^2}} =$$

$$= \frac{2\pi R_{earth}}{t_{earth}} \sqrt{\frac{R_{earth}}{R_{asteroid}}} = \frac{2\pi \cdot 150\ m \cdot 10^9}{365 \cdot 24 \cdot 60 \cdot 60\ s} \sqrt{\frac{150}{259}} \approx 22744 \frac{m}{s}$$

$$t_{asteroid} = \frac{2\pi R_{asteroid}}{\frac{2\pi R_{earth}}{t_{earth}} \sqrt{\frac{R_{earth}}{R_{asteroid}}}} = t_{earth} \frac{R_{asteroid}}{R_{earth}} \sqrt{\frac{R_{asteroid}}{R_{earth}}} = 1\ year \cdot \frac{259}{150} \sqrt{\frac{259}{150}} \approx$$

$$\approx 2.269\ years$$

Answer:

$t_{asteroid} \approx 2.269\ years$

$v_{asteroid} \approx 22744 \frac{m}{s}$