Why is not there any electric field inside uniformly charged sphere?

Solution.

We have spherical symmetry in problems.

Using the Gauss's law (Gauss's flux theorem). We have according this law

$$\oiint_S \vec{E} d\vec{A} = \frac{Q}{\varepsilon_0},$$

Where $d\vec{A}$ – is a vector representing an infinitesimal element of area, $\vec{E}$ – is the electric field, Q – is the total charge enclosed within closed surface ***S*** enclosing any volume ***V***.

Inside uniformly charged sphere we have $\vec{E} \parallel d\vec{A} \parallel \vec{r}$, $\vec{r}$ is radius-vector to point from center of sphere, closed surface ***S*** is sphere embedded inside charged sphere (radius r_0).

In volume ***V*** (enclosing any volume ***V***) we haven't any electric charge, and from here

$$\oiint_S \vec{E} d\vec{A} = 4\pi r^2 E(r) = \frac{Q}{\varepsilon_0} = 0, \;\; r < r_0$$

$$\Rightarrow E(r) = 0$$

Answer

$$E(r) = 0$$