

QUESTION:

The displacement of a block attached to a horizontal spring whose spring constant is 6 N/m is given by $x = 0.4 \cos(4.1t - 0.8)$ m. What is the earliest time ($t > 0$) when the kinetic energy equals one-half the potential energy?*** I saw the answer to this question but I didn't understand why the following was done: At the time when the potential energy equals to the half of it, the displacement of the block would be $x = \sqrt{2 \cdot 0.24/k} = 0.28$ m.

SOLUTION:

The potential energy is

$$E_p = \frac{k \cdot x^2}{2} = \frac{k \cdot (0.4 \cos(4.1t - 0.8))^2}{2}$$

The kinetic energy is

$$E_k = \frac{mv^2}{2}$$

$$v = \frac{dx}{dt} = -4.1 \cdot 0.4 \sin(4.1t - 0.8)$$

$$E_k = \frac{mv^2}{2} = \frac{m \cdot 1.64^2 \cdot \sin^2(4.1t - 0.8)}{2}$$

$$\text{Total energy is } E_{\text{tot}} = \frac{kx_m^2}{2} = 0.48 \text{ J}$$

So when the potential energy is equals to half of it

$$\frac{k \cdot x^2}{2} = \frac{0.48}{2} = 0.24$$

$$x = \sqrt{\frac{2 \cdot 0.24}{k}} = 0.283 \text{ m}$$