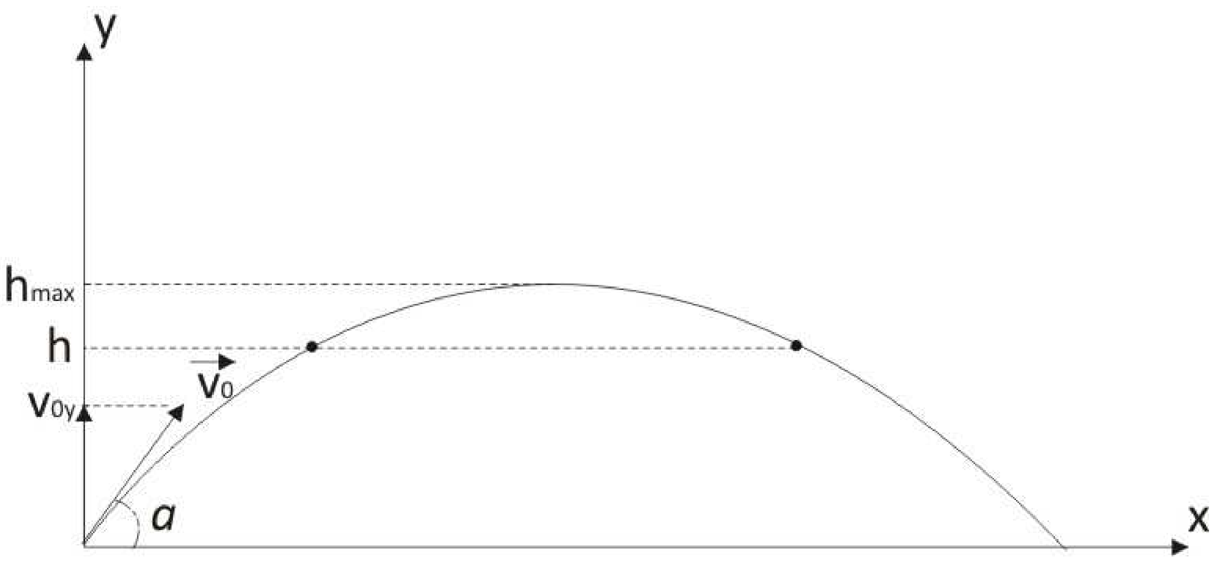
Imagine that you are solving a problem in projectile motion in which you are asked to find the time at which a projectile reaches a certain vertical position. When you solve the problem, you will find two different position values for time that both satisfy the condition of the problem. Explain how this result is not only possible, but also logical, remember to cite your sources.

Solution.

For example: a projectile moves with an initial velocity of $\upsilon = 100\frac{m}{s}$ at an elevation of $\alpha = 30°$.

First of all we find h_{max}.

A diagram $y = y(x)$.



$$\upsilon_y = \upsilon_0 sin\alpha.$$

$$h_{max} = \frac{\upsilon_y^2 - \upsilon_{0y}^2}{-2g}.$$

At the max height $\upsilon_y = 0$:

$$h_{max} = \frac{-\upsilon_{0y}^2}{-2g};$$

$$h_{max} = \frac{\upsilon_{0y}^2}{2g};$$

$$h_{max} = \frac{\upsilon_0^2 sin^2\alpha}{2g}.$$

$$h_{max} = \frac{100^2 \cdot sin^2 30°}{2 \cdot 9.8} \approx 127.55(m).$$

Then we find the time at which a projectile reaches the vertical position $h = 100m$.

$$h = \upsilon_{0y}t - \frac{gt^2}{2};$$

$$h = v_0 sin\alpha t - \frac{gt^2}{2};$$

$$\frac{gt^2}{2} - v_0 sin\alpha t + h = 0.$$

$$t^2 - \frac{2v_0 sin\alpha}{g} t + \frac{2h}{g} = 0.$$

$$t^2 - \frac{2 \cdot 100 \cdot sin30°}{9.8} t + \frac{2 \cdot 100}{9.8} = 0.$$

$$t^2 - 10.2t + 20.4 = 0.$$

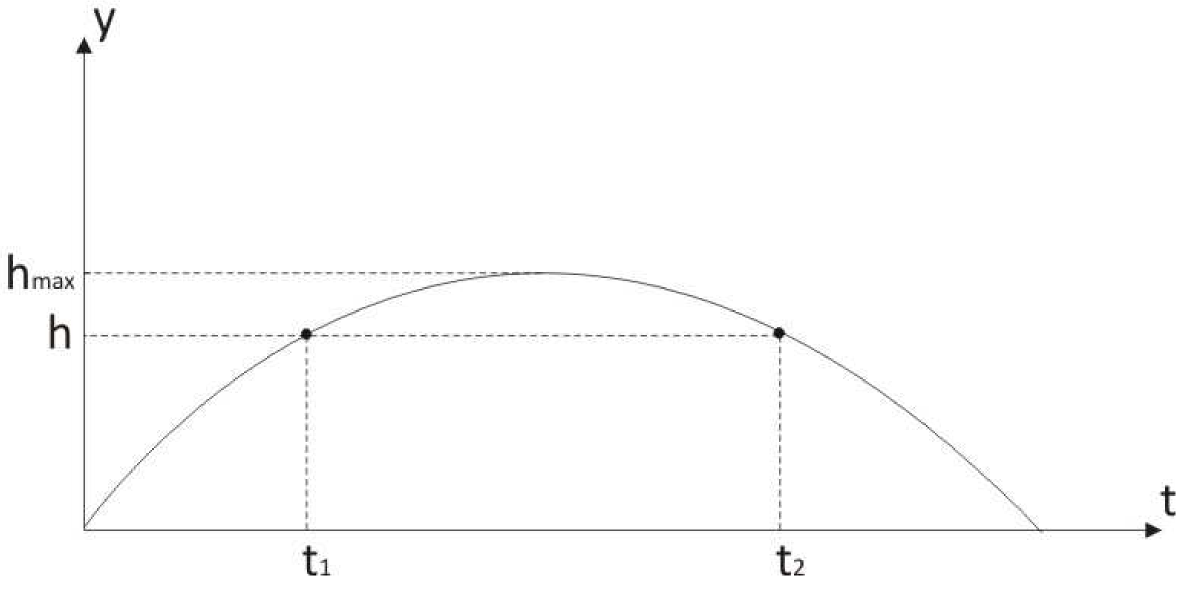
A discriminant:

$$\Delta = 10.2 - 4 \cdot 20.4 = 71.4;$$

$$t_1 = \frac{10.2 - \sqrt{71.4}}{2} = 0.88(s);$$

$$t_2 = \frac{10.2 + \sqrt{71.4}}{2} = 9.32(s).$$

A diagram $y = y(t)$.



$t_1 = 0.88s$ is the time when the projectile reaches the vertical position $h = 100m$ when it moves upward.

$t_2 = 9.32s$ is the time when the projectile reaches the vertical position $h = 100m$ when it moves downward.

This result is not only possible, but also logical.