QUESTION:

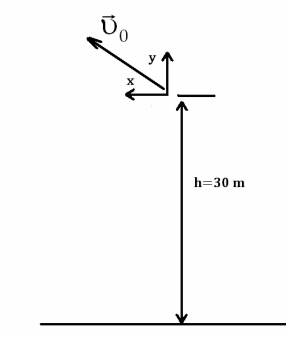
a man stands on the roof of a building that is 30m tall and throws a rock with a velocity of magnitude of 40m/s at an angle of 33 degress a bove the horizontal. Calculate the following:

a.) The maximum height above the roof reached by the rock.

b.) The magnitude of the velocity of the rock just before it strikes the ground.

c.) The horizontal distance from the base of the building to the point where the rock strikes the ground.

SOLUTION:



The equations of motion of a rock in chosen origin are:

$$x = \upsilon_0 \cos\alpha \cdot t$$

$$y = \upsilon_0 \sin\alpha \cdot t - \frac{g \cdot t^2}{2}$$

The rock's velocity projections:

$$\upsilon_x = \upsilon_0 \cos\alpha$$

$$\upsilon_y = \upsilon_0 \sin\alpha - gt$$

When a rock reaches the maximum height, the υ_y-projection of its velocity becomes zero, hence we can find the time t_m, it takes the rock to reach the maximum height:

$$0 = \upsilon_0 \sin\alpha - gt_m$$

$$t_m = \frac{\upsilon_0 \sin\alpha}{g}$$

Hence, the maximum height is

$$y_m = \upsilon_0 \sin\alpha \cdot t_m - \frac{g \cdot t_m^{\,2}}{2} = \upsilon_0 \sin\alpha \cdot \frac{\upsilon_0 \sin\alpha}{g} - \frac{g}{2}\left(\frac{\upsilon_0 \sin\alpha}{g}\right)^2 =$$

$$= \frac{\upsilon_0^2 \sin^2\alpha}{2g} \approx 24.2 \text{ m}$$

($\frac{\upsilon_0^2 \sin^2\alpha}{2g}$ - if this formula is known you can use it without derivation given above)

When rock strikes the ground, its y-coordinate is

$$-h = \upsilon_0 \sin\alpha \cdot t_s - \frac{g \cdot t_s^2}{2}$$

$$t_s^2 - \frac{2\upsilon_0 \sin\alpha}{g} t_s + \frac{2h}{g} = 0$$

$$t_{s,1} = \frac{\frac{2\upsilon_0 \sin\alpha}{g} + \sqrt{\left(\frac{2\upsilon_0 \sin\alpha}{g}\right)^2 - \frac{8h}{g}}}{2} = 5.55 \text{ s}$$

$$t_{s,2} = \frac{\frac{2\upsilon_0 \sin\alpha}{g} - \sqrt{\left(\frac{2\upsilon_0 \sin\alpha}{g}\right)^2 - \frac{8h}{g}}}{2} < 0 - \text{no physical sense}$$

The magnitude of the velocity of the rock just before it strikes the ground is:

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(\upsilon_0 \cos\alpha)^2 + (\upsilon_0 \sin\alpha - g \cdot t_s)^2} = 46.8 \text{ m/s}$$

The horizontal distance from the base of the building to the point where the rock strikes the ground:

$$x_{max} = \upsilon_0 \cos\alpha \cdot t_s = 186.2 \text{ m}$$

ANSWER:
The maximum height is 24.2 m

The magnitude of the velocity of the rock just before it strikes the ground is 46.8 m/s

The horizontal distance from the base of the building to the point where the rock strikes the ground is 186.2