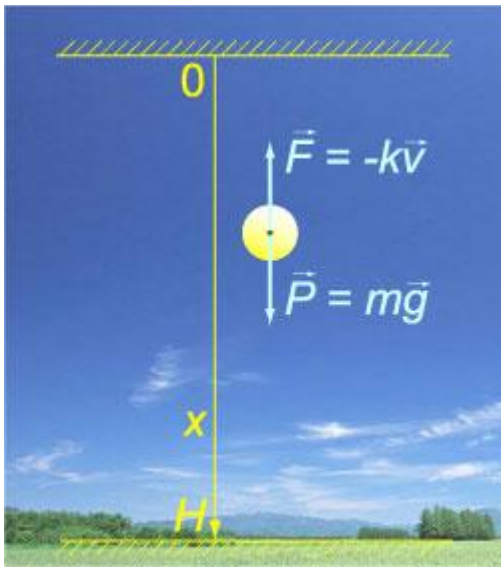
Task:

Does mass affect how an object speeds up under the pull of gravity e.g If two cars one with more mass and and one with less mass are pushed with the same force and the air resistance is also same on them because the cars are the same kind which one will speed up more? And of the two cars which one will reach the ground first if they are thrown from air?

Solution:



$$m\frac{d^2x}{dt^2} = mg - k\frac{dx}{dt}.$$

$$m\frac{dv}{dt} = mg - kv$$

$$\frac{dv}{dt} = g - \frac{k}{m}v.$$

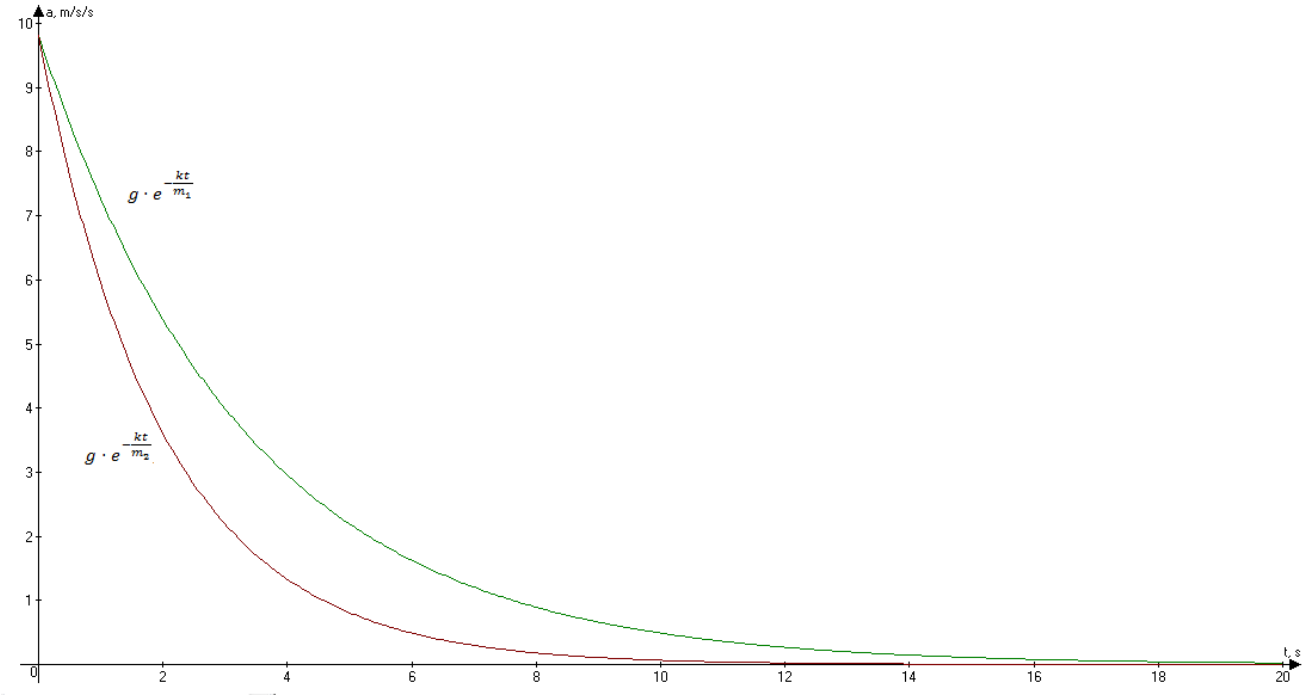
$$\frac{dv}{g - \frac{k}{m}v} = dt, \quad \Rightarrow \int_0^v \frac{du}{g - \frac{k}{m}u} = t.$$

$$-\frac{m}{k}\int_0^v \frac{d\left(g - \frac{k}{m}u\right)}{g - \frac{k}{m}u} = t, \quad \Rightarrow -\frac{m}{k}\left[\ln\left(g - \frac{k}{m}u\right)\right]\Big|_0^v = t, \quad \Rightarrow \ln\frac{g - \frac{k}{m}v}{g} = -\frac{k}{m}t, \quad \Rightarrow \ln\left(1 - \frac{k}{mg}v\right) = -\frac{k}{m}t,$$

$$\Rightarrow 1 - \frac{k}{mg}v = \exp\left(-\frac{k}{m}t\right), \quad \Rightarrow \frac{k}{mg}v = 1 - \exp\left(-\frac{k}{m}t\right), \quad \Rightarrow v(t) = \frac{mg}{k}\left[1 - \exp\left(-\frac{k}{m}t\right)\right].$$

$$a_1 = \frac{dv_1}{dt} = g \cdot e^{-\frac{kt}{m_1}}; \quad a_2 = \frac{dv_2}{dt} = g \cdot e^{-\frac{kt}{m_2}};$$

If $m_1 > m_2$



$a_1 > a_2$ at $t > 0$

Answer:

If $m_1 > m_2$

$a_1 > a_2$ at $t > 0$