Task:

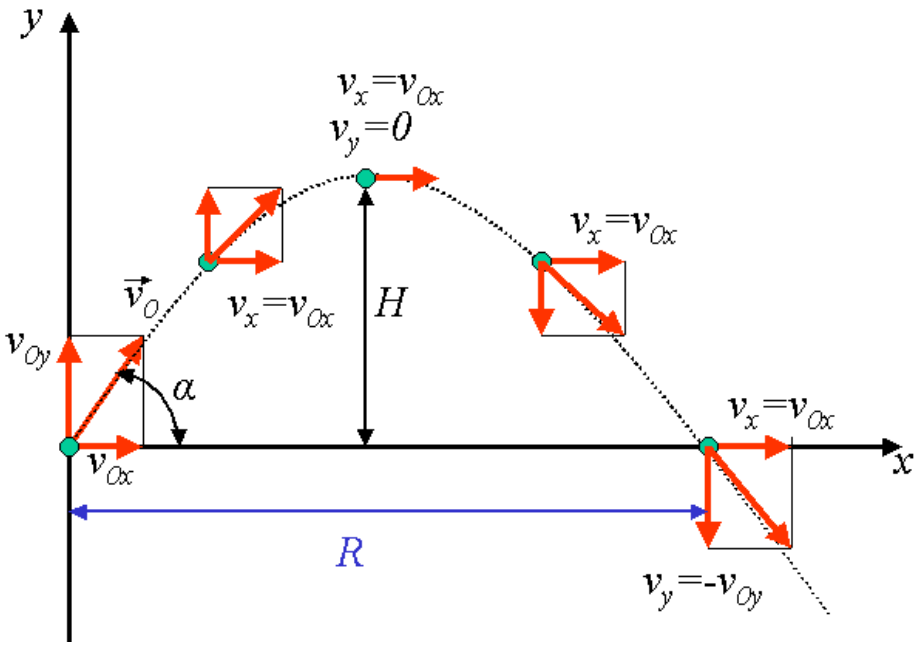
A football leaves the toe of a painter at 200 ft/sec. and strikes the ground 500 ft. away.

a.) Determine its initial angle of projection

b.) Find the total time of flight

c.) Find the maximum height reached.

Solution:



$$v_0 = 200\frac{ft}{s}$$

$$R = 500\ ft$$

$$R = v_0 \cos\alpha \cdot t$$

$$H = v_0 \sin\alpha \cdot \frac{t}{2} - g \cdot \frac{\left(\frac{t}{2}\right)^2}{2} = \frac{v_0^2 \sin^2\alpha}{2g}$$

$$\sin^2\alpha = \left(v_0 \sin\alpha \cdot \frac{t}{2} - g \cdot \frac{\left(\frac{t}{2}\right)^2}{2}\right) \cdot \frac{2g}{v_0^2} = \left(v_0 \sin\alpha \cdot t - g \cdot \left(\frac{t}{2}\right)^2\right) \cdot \frac{g}{v_0^2}$$

$$t = \frac{R}{v_0 \cos\alpha}$$

$$\sin^2\alpha = \left(v_0 \sin\alpha \cdot \frac{R}{v_0 \cos\alpha} - g \cdot \left(\frac{\frac{R}{v_0 \cos\alpha}}{2}\right)^2\right) \cdot \frac{g}{v_0^2} =$$

$$= \left(\sin\alpha \cdot \frac{R}{\cos\alpha} - g \cdot \left(\frac{R}{\frac{v_0 \cos\alpha}{2}} \right)^2 \right) \cdot \frac{g}{v_0^2} = \left(\frac{\sin\alpha \cdot R}{\cos\alpha} - \frac{g \cdot R^2}{4v_0^2 \cos^2\alpha} \right) \cdot \frac{g}{v_0^2} =$$

$$= \left(\frac{4v_0^2 \cdot \cos\alpha \cdot \sin\alpha \cdot R - g \cdot R^2}{4v_0^2 \cos^2\alpha} \right) \cdot \frac{g}{v_0^2} = \frac{g \cdot R}{4v_0^4 \cos^2\alpha} \cdot (4v_0^2 \cdot \cos\alpha \cdot \sin\alpha - g \cdot R)$$

$$\sin^2\alpha = \frac{g \cdot R}{4v_0^4 \cos^2\alpha} \cdot (4v_0^2 \cdot \cos\alpha \cdot \sin\alpha - g \cdot R)$$

$$\alpha_1 = 11.85°, \qquad \alpha_2 = 78.15°$$

$$t_1 = \frac{R}{v_0 \cos\alpha_1} = 2.554\ s$$

$$t_2 = \frac{R}{v_0 \cos\alpha_2} = 12.174\ s$$

$$H_1 = \frac{v_0^2 \sin^2\alpha_1}{2g} = 26.232\ ft$$

$$H_2 = \frac{v_0^2 \sin^2\alpha_2}{2g} = 595.852\ ft$$

Answer:

a) $\alpha_1 = 11.85°,\ \alpha_2 = 78.15°$

b) $t_1 = \frac{R}{v_0 \cos\alpha_1} = 2.554\ s$

$$t_2 = \frac{R}{v_0 \cos\alpha_2} = 12.174\ s$$

c) $H_1 = \frac{v_0^2 \sin^2\alpha_1}{2g} = 26.232\ ft$

$$H_2 = \frac{v_0^2 \sin^2\alpha_2}{2g} = 595.852\ ft$$