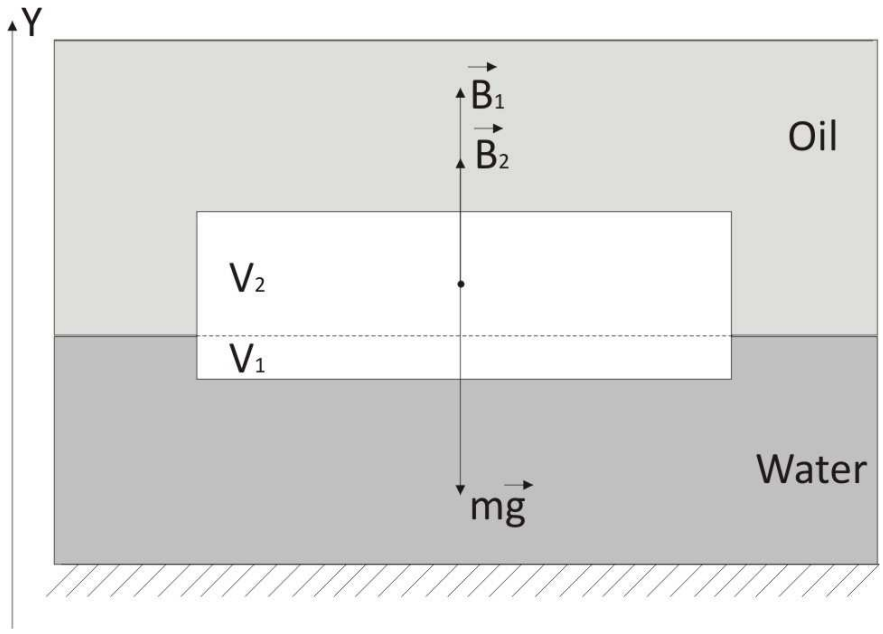
A block of wood having specific gravity 0.85 floats on water. Some oil of specific gravity 0.82 is poured on the surface of water until the wooden block is just immersed. Calculate the fraction of block lying below the surface of water in the second case.

Solution.

$$\frac{\rho_1}{\rho_w} = 0.85, \frac{\rho_2}{\rho_w} = 0.82;$$

$$\frac{V_1}{V} - ?$$



Newton's second law in vector form:

$$m\vec{a} = \overrightarrow{B_1} + \overrightarrow{B_2} + m\vec{g}.$$

A block is at rest, then:

$$\vec{a} = 0.$$

$$0 = \overrightarrow{B_1} + \overrightarrow{B_2} + m\vec{g}.$$

Projection on Y:

$$0 = B_1 + B_2 - mg;$$

$$B_1 + B_2 = mg.$$

B_1 – the buoyancy force in the water.

B_2 – the buoyancy force in the oil.

m - the mass of a block.

$$m = \rho_1 V;$$

ρ_1 – the density of the wood.

V – the volume of the block.

$$B_1 = \rho_w V_1 g;$$

$$B_2 = \rho_2 V_2 g.$$

V_2 - the part of volume of the block above water level.

$$V_2 = V - V_1;$$

$$B_2 = \rho_2 (V - V_1) g;$$

ρ_w – the density of the water;

ρ_2 – the density of the oil;

V_1 – the part of volume of the block below water level.

$$\rho_w V_1 g + \rho_2 (V - V_1) g = \rho_1 V g.$$

Divide by g:

$$\rho_w V_1 + \rho_2 (V - V_1) = \rho_1 V.$$

Divide by ρ_w:

$$V_1 + \frac{\rho_2}{\rho_w}(V - V_1) = \frac{\rho_1}{\rho_w} V;$$

$$V_1 + \frac{\rho_2}{\rho_w} V - \frac{\rho_2}{\rho_w} V_1 = \frac{\rho_1}{\rho_w} V.$$

Divide by V:

$$\frac{V_1}{V} + \frac{\rho_2}{\rho_w} - \frac{\rho_2}{\rho_w}\frac{V_1}{V} = \frac{\rho_1}{\rho_w};$$

$$\frac{V_1}{V}\left(1 - \frac{\rho_2}{\rho_w}\right) = \frac{\rho_1}{\rho_w} - \frac{\rho_2}{\rho_w};$$

$$\frac{V_1}{V} = \frac{\frac{\rho_1}{\rho_w} - \frac{\rho_2}{\rho_w}}{\left(1 - \frac{\rho_2}{\rho_w}\right)}.$$

$$\frac{V_1}{V} = \frac{0.85 - 0.82}{(1 - 0.82)} = 0.17.$$

Answer: $\frac{V_1}{V} = 0.17.$