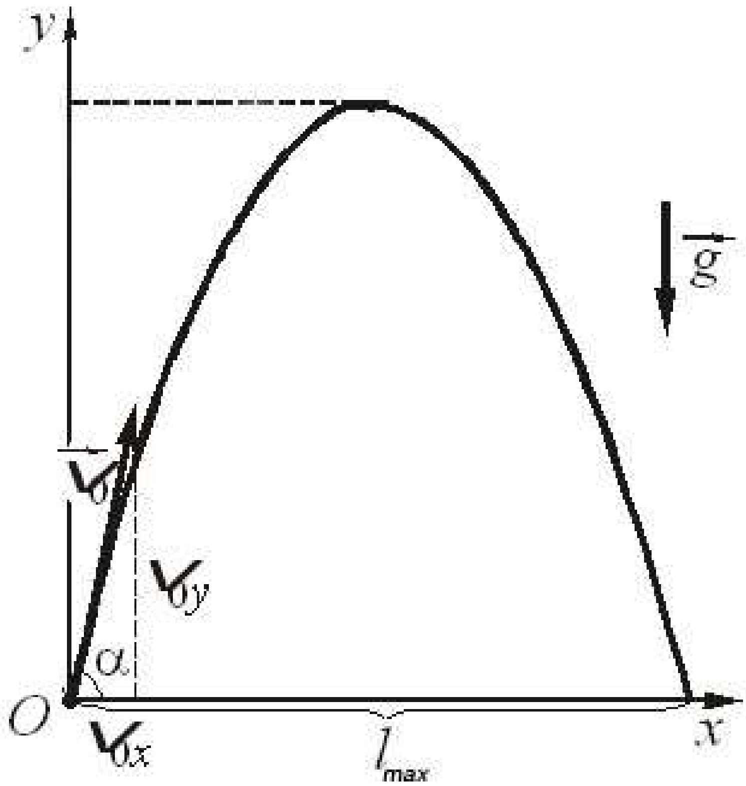
The horizontal distance covered by a projectile in motion is maximized at which angle?

Solution.



$$v_x = v_0 cos\alpha;$$

$$v_y = v_0 sin\alpha.$$

The max distance.

$$l = v_{0x} t;$$

$$l = v_0 cos\alpha t;$$

The time of flight.

$$y = v_{0y} t - \frac{gt^2}{2};$$

$$y = v_0 sin\alpha t - \frac{gt^2}{2}.$$

At the end of the flight $y = 0$:

$$0 = v_0 sin\alpha t - \frac{gt^2}{2};$$

$$v_0 sin\alpha t = \frac{gt^2}{2};$$

$$v_0 sin\alpha = \frac{gt}{2};$$

$$t = \frac{2v_0 sin\alpha}{g}.$$

The max distance.

$$l = v_0 cos\alpha \frac{2v_0 sin\alpha}{g};$$

$$l = \frac{2v_0^2 sin\alpha cos\alpha}{g}.$$

When the distance is maximized - the derivative of a function of the distance is zero (an angle is the independent variable):

$$l'_{max} = 0;$$

$$\left(\frac{2v_0^2 sin\alpha cos\alpha}{g}\right)' = \frac{2v_0^2}{g}\left((sin\alpha)' cos\alpha - sin\alpha(cos\alpha)'\right) =$$

$$= \frac{2v_0^2}{g}(cos\alpha cos\alpha - sin\alpha sin\alpha) = \frac{2v_0^2}{g}(cos^2\alpha - sin^2\alpha);$$

$$\frac{2v_0^2}{g}(cos^2\alpha - sin^2\alpha) = 0;$$

$$cos^2\alpha = sin^2\alpha;$$

$$tan^2\alpha = 1;$$

$$tan\alpha = 1;$$

$$arctan1 = 45°;$$

$$\alpha = 45°.$$

Answer:

The horizontal distance covered by a projectile in motion is maximized at $\alpha = 45°$.