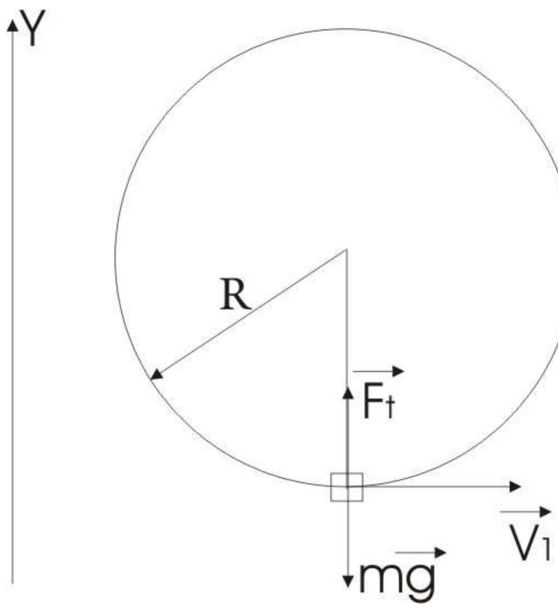
A bucket of mass 1.40 kg is whirled in a vertical circle of radius 1.10 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N.
a) Find the speed of the bucket.
b) How fast must the bucket move at the top of the circle so that the rope does not go slack?

Solution.

$$m = 1.40kg, r = 1.10m, F_t = 25.0N;$$

$$\boldsymbol{v_1}-?\ \boldsymbol{v_2}-?$$

a) Find the speed of the bucket.



Newton's second law in vector form:

$$m\overrightarrow{a_1} = \overrightarrow{F_{t1}} + m\vec{g}.$$

Projection on OY:

$$ma_1 = F_{t1} - mg;$$

The acceleration due to change in the direction is:

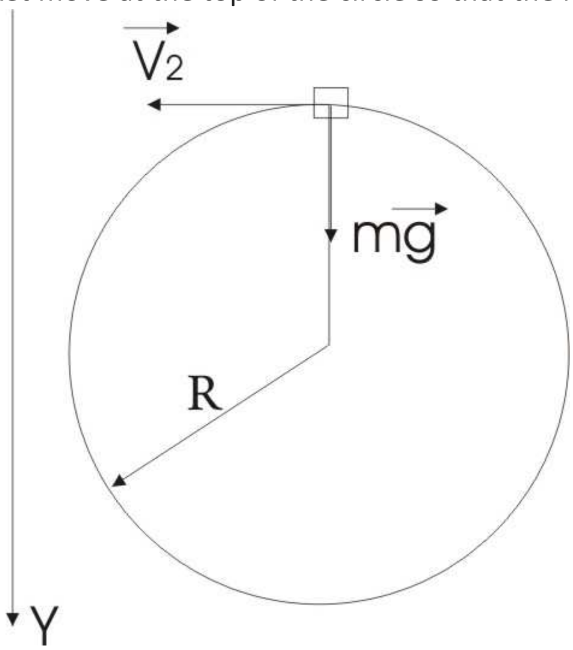
$$a_1 = \frac{v_1^2}{r};$$

$$m\frac{v_1^2}{r} = F_{t1} - mg;$$

$$v_1 = \sqrt{r\left(\frac{F_{t1}}{m} - g\right)};$$

$$v_1 = \sqrt{1.10\left(\frac{25}{1.40} - 9.8\right)} = 2.98\left(\frac{m}{s}\right).$$

b) How fast must the bucket move at the top of the circle so that the rope does not go slack?



Newton's second law in vector form:

$$m\vec{a_2} = \vec{F_{t2}} + m\vec{g}.$$

Projection on OY:

$$ma_2 = F_{t2} + mg;$$

$$F_{t2} = 0;$$

$$ma_2 = mg;$$

$$a_2 = g;$$

The acceleration due to change in the direction is:

$$a_2 = \frac{v_2^2}{r};$$

$$v_2 = \sqrt{rg}.$$

$$v_2 = \sqrt{1.10 \cdot 9.8} = 3.28\left(\frac{m}{s}\right).$$

Answer:

a) $v_1 = 2.98\frac{m}{s}$.
b) $v_2 = 3.28\frac{m}{s}$.