Task:

A string of natural length L extends to a new length L_1 under tensile force F. If Hooke's Law applies, what is the work done in stretching the spring?

Solution:

A horizontal spring exerts a force ***F***=(*kx*, 0, 0) that is proportional to its deflection in the *x* direction. The work of this spring on a body moving along the space curve

s(t) = (*x*(t), *y*(t), *z*(t)), is calculated using its velocity, ***v***=(v_x, v_y, v_z), to obtain

$$W = \int_0^t \boldsymbol{F} \cdot \boldsymbol{v} dt = \int_0^t kxv_x dt = \frac{1}{2}kx^2.$$

For convenience, consider contact with the spring occurs at *t* = 0, then the integral of the product of the distance *x* and the *x*-velocity, xv_x, *is* $(1/2)x^2$.

The function $U(x)= 1/2\ kx^2$ is called the potential energy of a linear spring.

Given:

$x = L_1 - L,$

$W = \frac{1}{2}k(L_1 - L)^2$

Answer:

$W = \frac{1}{2}k(L_1 - L)^2$