

A train starts from rest from a station with acceleration 0.2 m/s^2 on a straight track and then comes to rest after attaining maximum velocity on another station due to retardation of 0.4 m/s^2 . If total time spent is half an hour, then distance between two stations is.

Solution.

$$a_1 = 0.2 \frac{\text{m}}{\text{s}^2}, a_2 = 0.4 \frac{\text{m}}{\text{s}^2}, t = 30 \text{ min} = 1800 \text{ s};$$

$$s = ?$$

A displacement of the train, when it went with acceleration a_1 :

$$s_1 = \frac{v^2 - v_0^2}{2a_1}.$$

$v_0 = 0$, v - attaining maximum velocity.

A displacement of the train, when it went with retardation a_2 :

$$s_2 = \frac{u^2 - u_0^2}{2a_2}.$$

$u = 0$, u_0 - attaining maximum velocity.

A final velocity, when the train goes with acceleration a_1 is a initial velocity, when it goes with retardation a_2 .

$u_0 = v$, then:

$$s_1 = \frac{v^2}{2a_1};$$

$$s_2 = \frac{v^2}{2a_2}.$$

Divide first equation by second:

$$\frac{s_1}{s_2} = \frac{v^2 2a_2}{2a_1 v^2} = \frac{a_2}{a_1};$$

$$\frac{s_1}{s_2} = \frac{0.4}{0.2} = 2;$$

$$\frac{s_1}{s_2} = 2.$$

A velocity of the train, when it went with acceleration a_1 :

$$v = v_0 + a_1 t_1;$$

$$v_0 = 0,$$

$$v = a_1 t_1.$$

A velocity of the train, when it went with retardation a_2 :

$$u = u_0 - a_2 t_2;$$

$$u = 0;$$

$$u_0 = a_2 t_2$$

$$u_0 = v;$$

$$a_1 t_1 = a_2 t_2;$$

$$\frac{t_1}{t_2} = \frac{a_2}{a_1};$$

$$\frac{t_1}{t_2} = \frac{0.4}{0.2} = 2;$$

$$\frac{t_1}{t_2} = 2.$$

A displacement of the train (is the distance between two stations):

$$s = s_1 + s_2;$$

$$s_2 = \frac{s_1}{2};$$

$$s = s_1 + \frac{s_1}{2} = \frac{3}{2} s_1.$$

A displacement of the train, when it went with acceleration a_1 :

$$s_1 = v_0 + \frac{a_1 t_1^2}{2};$$

$v_0 = 0$ then:

$$s_1 = \frac{a_1 t_1^2}{2}.$$

The total time:

$$t = t_1 + t_2;$$

$$t_2 = \frac{t_1}{2};$$

$$t = t_1 + \frac{t_1}{2} = \frac{3}{2} t_1;$$

$$t_1 = \frac{2}{3}t;$$

$$s = \frac{3}{2}s_1 = \frac{3}{2} \frac{a_1 t_1^2}{2} = \frac{3}{2} \frac{a_1}{2} \left(\frac{2}{3}t\right)^2 = \frac{3a_1 4t^2}{4 \cdot 9} = \frac{a_1 t^2}{3};$$

$$s = \frac{a_1 t^2}{3}.$$

$$s = \frac{0.2(1800)^2}{3} = 216000(m);$$

$$s = 216km.$$

Answer: $s = 216km$.