Task:

A train at rest starts moving with uniform acceleration, keeps moving for 2 min. It reaches 48 km/h after 5 min of its uniform retardation and stops after 3 min. How much distance it covered?

Solution:

Given:

$$t_{acc} = 2 \min = \frac{1}{30} h$$

$$t_{ret} = 5 \min = \frac{1}{12}h$$

$$t_{stop} = 3 \min = \frac{1}{20} h$$

$$\dot{s}_2(t_{ret}) = 48 \ km/h$$

$$\dot{s}_2(t_{ret} + t_{stop}) = 0 \ km/h$$

$$s_1(t) = \frac{a_1 \cdot t^2}{2}$$

$$s_2(t_1) = \frac{a_2 \cdot t_1^2}{2} + \dot{s}_1(t_{acc}) \cdot t_1$$

$$s_{total} = s_1(t_{acc}) + s_2(t_{ret} + t_{stop})$$

$$s_1(t_{acc}) = \frac{a_1 \cdot t_{acc}^2}{2}$$

$$\dot{s}_1(t) = a_1 \cdot t$$

$$\dot{s}_1(t_{acc}) = a_1 \cdot t_{acc}$$

$$\dot{s}_2(t_1) = a_2 \cdot t_1 + \dot{s}_1(t_{acc}) = a_2 \cdot t_1 + a_1 \cdot t_{acc}$$

$$\dot{s}_2(t_{ret}) = a_2 \cdot t_{ret} + a_1 \cdot t_{acc} = 48 \ km/h$$

$$\dot{s}_2(t_{ret} + t_{stop}) = a_2 \cdot (t_{ret} + t_{stop}) + a_1 \cdot t_{acc} = 0 \, km/h$$

$$a_1 = 3840 \ km/h^2$$
, $a_2 = -960 \ km/h^2$;

$$s_{total} = \frac{a_1 \cdot t_{acc}^2}{2} + \frac{a_2 \cdot (t_{ret} + t_{stop})^2}{2} + a_1 \cdot t_{acc} \cdot (t_{ret} + t_{stop}) = \frac{32}{3} \ km$$

Answer:

The train covered $\frac{32}{3}$ km