A closed single loop of wire is moving to the right in a magnetic field of flux density $B = 0.8T$. The field is perpendicular to the plane of the coil in the shaded region shown in the diagram. The dimensions of the loop are $50cm \times 60cm$. The loop is moving at a constant speed of $v = 4ms^{-1}$ and its resistance is $R = 20\Omega$.

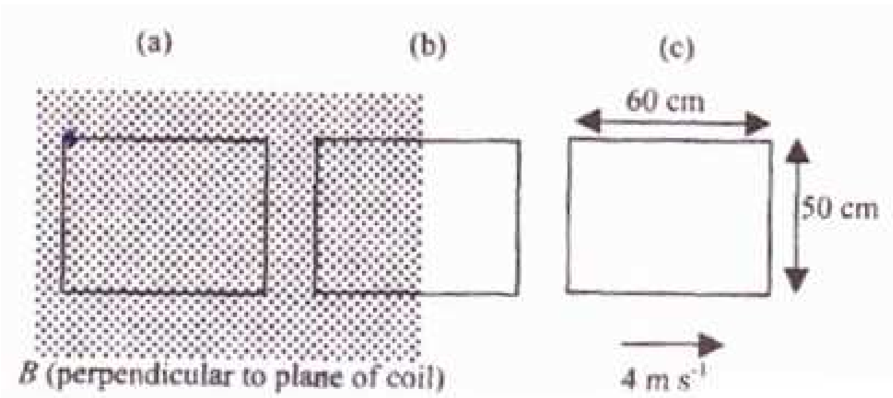
1) Explain why no current flows in the loop when the loop is in the positions (a) and (c).
2) Calculate the current flowing in the loop when it is in position (b).
3) Calculate the work required to move the coil from the position (a) to the position (c).

Solution.

1). Faraday's law of induction
Electromotive force (EMF) produced around a closed path is proportional to the rate of change of the magnetic flux through any surface bounded by that path: $\mathcal{E} = -\frac{d\Phi}{dt}$. This means that an electric current will be induced in any closed circuit when the magnetic flux through a surface bounded by the conductor changes. From Ohm's law: $I = \frac{U}{R}$; $U = \mathcal{E}$; $I = \frac{\mathcal{E}}{R}$; $I = -\frac{1}{R}\frac{d\Phi}{dt}$;
In the positions (a) no current flows, because the magnetic flux through the closed single loop is constant, magnetic flux: $\Phi = BScos\theta$, $B = const$ $(B = 0.8T)$, area of the plane of the closed single loop $S = const$ $(S = a \cdot b)$, $\Phi = const$, then $\frac{d\Phi}{dt} = 0$, $I = 0$.
In the positions (c) the magnetic flux is zero, because there is not magnetic field $B = 0$, then $\Phi = 0$ it does not change, then no current flows $I = 0$.

2). $B = 0.8T$, $a = 50cm = 0.5m$, $b = 60cm = 0.6m$, $v = 4ms^{-1} = 4\frac{m}{s}$, $R = 20\Omega$.

$I-?$



From Ohm's law:

$$I = \frac{U}{R};$$
$$U = \mathcal{E};$$
$$I = \frac{\mathcal{E}}{R};$$

From Faraday's law of induction:

$$\mathcal{E} = -\frac{d\Phi}{dt};$$
$$I = -\frac{1}{R}\frac{d\Phi}{dt}.$$

The magnetic flux changes uniformly, because the loop is moving at a constant speed, then:

$$I = -\frac{1}{R}\frac{\Delta\Phi}{\Delta t}.$$
$$\Delta\Phi = \Phi_2 - \Phi_1.$$

Φ_1 - the magnetic flux, when the loop is in the positions (a);
Φ_2 - the magnetic flux, when the loop is in the positions (c).
$\Phi_1 = BScos\theta = BS$, because the field is perpendicular to the plane of the coil and $\theta = 0^o$ $cos\theta$=1. θ is the angle between the magnetic field lines and the normal (perpendicular) to S.
$\Phi_2 = 0$, because $B = 0$.

$$I = -\frac{1}{R}\frac{\Phi_2 - \Phi_1}{\Delta t};$$
$$I = -\frac{1}{R}\frac{(-\Phi_1)}{\Delta t};$$
$$I = \frac{1}{R}\frac{\Phi_1}{\Delta t} = \frac{BS}{R\Delta t};$$
$$S = ab;$$
$$I = \frac{Bab}{R\Delta t};$$
$$b = v\Delta t;$$
$$\Delta t = \frac{b}{v};$$
$$I = \frac{Bav}{R}.$$
$$I = \frac{0.8 \cdot 0.5 \cdot 4}{20} = 0.08(A).$$

3). The work required to move the coil from the position (a) to the position (c) is equal to the current work.

$$A = P\Delta t$$

$P = UI$ or $P = I^2R$ or $P = \frac{U^2}{R}$.
We use $P = \frac{U^2}{R}$.

$$A = P\Delta t;$$
$$A = \frac{U^2}{R}\Delta t;$$
$$A = \frac{\mathcal{E}^2}{R}\Delta t.$$

From Faraday's law of induction:

$$A = \left(\frac{\Delta\Phi}{\Delta t}\right)^2 \frac{\Delta t}{R};$$

From the question 2) we use $\Delta\Phi$:

$$A = \left(\frac{\Phi_2 - \Phi_1}{\Delta t}\right)^2 \frac{\Delta t}{R};$$

$$A = \left(\frac{\Phi_1}{\Delta t}\right)^2 \frac{\Delta t}{R};$$
$$A = \frac{B^2 S^2}{\mathrm{R}\Delta t};$$
$$A = \frac{B^2 a^2 b^2}{\mathrm{R}\Delta t};$$

From the question 2) we use Δt:

$$A = \frac{B^2 a^2 b^2 v}{Rb};$$
$$A = \frac{B^2 a^2 b v}{R}.$$
$$A = \frac{0.8^2 \cdot 0.5^2 \cdot 0.6 \cdot 4}{20} = 0.0192(J) = 19.2(mJ).$$

Answer:

1). In the positions (a) no current flows, because the magnetic flux through the closed single loop is constant, magnetic flux: $\Phi = BS$, $B = const$ $(B = 0.8T)$, area of the plane of the closed single loop $S = const$ $(S = a \cdot b)$, $\Phi = const$; $\frac{d\Phi}{dt} = 0$, $I = 0$.
In the positions (c) magnetic flux is zero, because there is not magnetic field $B = 0$, then $\Phi = 0$ it does not change, then no current flows $I = 0$.

2). $I = 0.08A$.

3). $A = 19.2mJ$.