Question

Given:

$m_1 = 24.0 \text{ g}$
$m_2 = 100 \text{ g}$
$l = 1.10 \text{ m}$
$\alpha = 52.0°$
$V_2 = 0$

Need to find: initial velocity V_1.

Solution:

At first we can find final velocity v_2 of the 100 g ball:

$\frac{m_2 \cdot v_2^2}{2} = m_2 \cdot g \cdot h = m_2 \cdot g \cdot l \cdot (1 - \cos\alpha) \Rightarrow$ we can find final velocity and it equal to

$v_2^2 = 2 \cdot g \cdot l \cdot (1 - \cos\alpha) \Rightarrow v_2 = \sqrt{2 \cdot g \cdot l \cdot (1 - \cos\alpha)}$. We will get:

$v_2 = \sqrt{2 \cdot g \cdot l \cdot (1 - \cos\alpha)} = \sqrt{2 \cdot 9.81 \cdot 1.10 \cdot (1 - \cos 52°)} = 2.88 \ \frac{m}{s}$.

Applying conservation of momentum principle, we get:

$m_1 \cdot V_1 + m_2 \cdot V_2 = m_1 \cdot v_1 + m_2 \cdot v_2 \Rightarrow |V_2 = 0| \Rightarrow v_1 = \frac{m_1 \cdot V_1 - m_2 \cdot v_2}{m_1} = V_1 - v_2 \cdot \frac{m_2}{m_1}$. Also applying principle of kinetic energy conservation, we get:

$$\frac{1}{2} \cdot m_1 \cdot V_1^2 + \frac{1}{2} \cdot m_2 \cdot V_2^2 = \frac{1}{2} \cdot m_1 \cdot v_1^2 + \frac{1}{2} \cdot m_2 \cdot v_2^2 \Rightarrow \left| \begin{matrix} V_2 = 0 \\ v_1 = V_1 - v_2 \cdot \frac{m_2}{m_1} \end{matrix} \right| \Rightarrow$$

$$\Rightarrow m_1 \cdot V_1^2 = m_1 \cdot V_1^2 + m_1 \cdot v_2^2 \cdot \frac{m_2^2}{m_1^2} - 2 \cdot m_1 \cdot V_1 \cdot v_2 \cdot \frac{m_2}{m_1} + m_2 \cdot v_2^2 \Rightarrow$$

$$\Rightarrow v_2^2 \cdot \frac{m_2^2}{m_1} + m_2 \cdot v_2^2 = 2 \cdot V_1 \cdot v_2 \cdot m_2 \Rightarrow V_1 = \frac{v_2^2 \cdot m_2^2 + m_2 \cdot m_1 \cdot v_2^2}{2 \cdot m_1 \cdot v_2 \cdot m_2} = \frac{v_2 \cdot (m_2 + m_1)}{2 \cdot m_1}.$$

So, we have the formula: $V_1 = \frac{v_2 \cdot (m_2 + m_1)}{2 \cdot m_1}$. We will get: $V_1 = \frac{2.88 \cdot (0.1 + 0.024)}{2 \cdot 0.024} = 7.44 \ \frac{m}{s}$.

Answer: $7.44 \ \frac{m}{s}$.