

Question: In a Young's double slit experiment a laser of wavelength 600 nm is used to obtain a pattern with a distance between the center bright fringe and the first bright fringe of 0.5 mm. A thin plate of glass (thickness of 100 micrometers and refractive index of 1.5) is then placed over one of the slits. What is the lateral displacement of the central fringe on the screen?

Answer:

We are given:

$$\lambda = 600 \text{ nm}$$

$$w_1 = 0.5 \text{ mm}$$

$$\Delta p = 100 \text{ } \mu\text{m}$$

$$n_1 = 1.5$$

In a Young's double slit experiment the spacing of the fringes at a distance z from the slits is given by

$$w = \frac{z\lambda}{d}, n = 0,1,2, \dots$$

where λ is the wavelength of the light.

Thus, for first case:

$$w_1 = 0.5 \text{ mm}$$

so:

$$d = \frac{z\lambda}{w_1}$$

In the second case optical path between slits and plane is increased:

$$p_2 = p - \Delta p + \Delta p * n_1 = d\theta + \Delta p(n_1 - 1)$$

The interference fringe maxima occur at angles

$$d \sin \theta + \Delta p(n_1 - 1) = n\lambda, n = 0,1,2,3$$

For small angles:

$$\sin \theta = \theta = \tan \theta$$

For central fringe $n = 0$, thus :

$$\theta = -\frac{\Delta p(n_1 - 1)}{d}$$

And

$$\tan \theta = \frac{y}{z} \approx \theta$$

So:

$$\frac{y_1}{z} = -\frac{\Delta p(n_1 - 1)}{d} = -\frac{\Delta p(n_1 - 1)}{\frac{z\lambda}{w_1}}$$

$$\frac{y_1}{z} = -\frac{\Delta p(n_1 - 1)w_1}{z\lambda}$$

$$y_1 = -\frac{\Delta p(n_1 - 1)w_1}{\lambda}$$

Calculating:

$$y_1 = -\frac{\Delta p(n_1 - 1)w_1}{\lambda} = \frac{100 * 10^{-6} * (1.5 - 1) * 0.5 * 10^{-3}}{600 * 10^{-9}} = 0.0416 \text{ m} = \mathbf{4.16 \text{ cm}}$$

See: <http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/slits.html>