A command post O monitors the movement of two of its ships in the Gulf. At 1200 hrs a battleship (B) has position (-2**i** + 10**j**) km relative to O and has constant velocity of (3**i** + 2**j**) kmh^{-1}. A frigate (F) is at the point with position vector (4**i** + 5**j**) km and has constant velocity (-3**i** + 7**j**) kmh^{-1}, where **i** and **j** are unit vectors directed due east and due north respectively.

a) The captain of one ship has been taken ill, show that the two ships will collide.

The command post contacts the battleship and orders it to reduce its speed to move with velocity (2**i** + 2**j**) kmh^{-1}.

b) Find an expression for the vector $\overrightarrow{BF}$ at time t hours after noon.

c) Find the distance between B and F at 1400 hrs.

d) Find the time at which F will be due north of B.

Solution

The position of each ship is given by it's position vector:

position = initial position + (velocity × time)

So for the battleship:

$$r_b = (-2\boldsymbol{i} + 10\boldsymbol{j}) + t(3\boldsymbol{i} + 2\boldsymbol{j})$$

And for the frigate:

$$r_f = (4\boldsymbol{i} + 5\boldsymbol{j}) + t(-3\boldsymbol{i} + 7\boldsymbol{j})$$

If the two ships are to collide then for some value of t their respective **i** and **j** components must be equal.

Therefore by equating **i**'s:

$$-2 + 3t = 4 - 3t$$

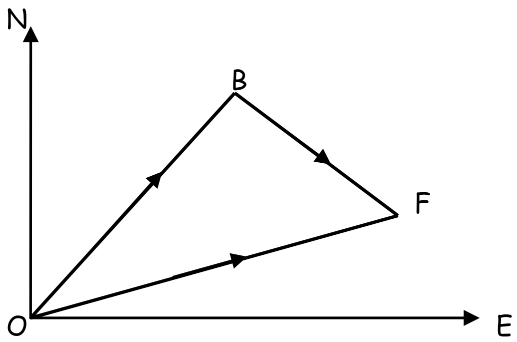
$$t = 1$$

Substituting the value of t = 1 into r_b and r_b gives the same position vector of (**i** + 12**j**). Therefore the two ships will collide after one hour at the point with position vector (**i** + 12**j**).

b) The position vector for the battleship must change to take account of the new velocity:

$$r_b = (-2\boldsymbol{i} + 10\boldsymbol{j}) + t(2\boldsymbol{i} + 2\boldsymbol{j})$$

We have been asked to find the vector $\overline{BF}$ as shown in the diagram below.



By triangle law:

$$\overline{OB} + \overline{BF} = \overline{OF}$$

$$\overline{BF} = \overline{OF} - \overline{OB}$$

Where $\overline{OB} = r_b$ and $\overline{OF} = r_f$

Therefore: $\overline{BF} = r_f - r_b = (4\mathbf{i} + 5\mathbf{j}) + t(-3\mathbf{i} + 7\mathbf{j}) - ((-2\mathbf{i} + 10\mathbf{j}) + t(2\mathbf{i} + 2\mathbf{j}))$

$$\overline{BF} = (6\mathbf{i} - 5\mathbf{j}) + t(-5\mathbf{i} + 5\mathbf{j})$$

c) The magnitude of gives the distance between the two ships, at 1400 hrs, t = 2

$$\overline{BF} = (6\boldsymbol{i} - 5\boldsymbol{j}) + 2(-5\boldsymbol{i} + 5\boldsymbol{j}) = (-4\boldsymbol{i} + 5\boldsymbol{j})$$

Magnitude $=\sqrt{41}Km$

d) If F is due north of B, then $\overline{BF}$ will have no **i** component.

$$\overline{BF} = (6\mathbf{i} - 5\mathbf{j}) + t(-5\mathbf{i} + 5\mathbf{j})$$

$$6\boldsymbol{i} - 5t\boldsymbol{i} = 0$$

$$t = 1hr\ 12mins$$