As a prank, someone has balanced a pumpkin near the highest point of a grain silo. The silo is topped with a hemispherical cap that is frictionless when wet. The line from the center of curvature of the cap to the pumpkin makes an angle θi = 3.2° with the vertical. While you happen to be standing nearby in the middle of a rainy night, a breath of wind makes the pumpkin start sliding downward from rest. It loses contact with the cap when the line from the center of the hemisphere to the pumpkin makes a certain angle with the vertical. What is this angle, to the nearest 0.1°?

Solution:

Use the conservation of energy

$mgR\ cos\ (\theta i) = \frac{1}{2}mv^2 + mgRcos\ (\theta)$ or, $mgR[cos(\theta i) - cos(\theta)] = \frac{1}{2}mv^2$,

where, R is the radius of the cap, θ is the angle the pumpkin makes with the vertical. The origin of the potential energy is set at the center of the cap. When, the pumpkin loses contact, the component of the gravity can no longer provide the centripetal force, which is perpendicular to the surface.

This component of the gravity is calculated as $mgcos(\theta)$.

The centripetal force is $\frac{mv^2}{R}$, thus $\frac{mv^2}{R} = mgcos(\theta)$.

Now combine $mgR[cos(\theta i) - cos(\theta)] = \frac{1}{2}mv^2$ and $\frac{mv^2}{R} = mgcos(\theta)$, we have $cos(\theta) = \frac{2cos(\theta i)}{3}$.

So

$\theta = arccos\left[\frac{2cos(\theta i)}{3}\right] = arccos\left[\frac{2cos(3.2)}{3}\right] = 48{,}3°$

Answer: $\boldsymbol{\theta = 48{,}3°}$.