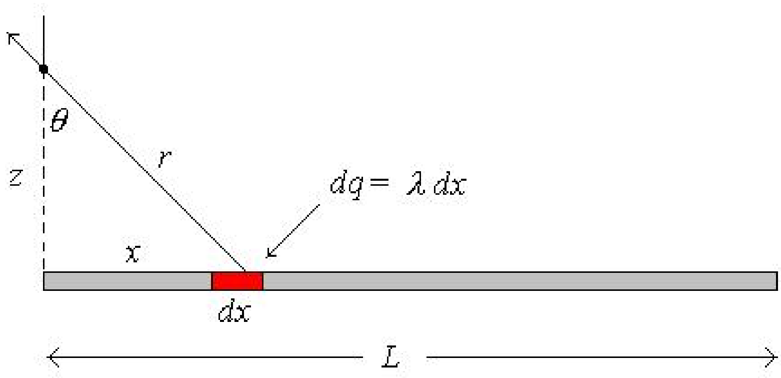
Find the electric field intensity at a distance z above one end of the straight line segment of length l ,which carries a uniform line charge k. Check that your formula is consistant what you would expect for z.>>>l.

Solution

Uniform line charge k = λ; length l=L.



$$E_Z = \frac{1}{4\pi\varepsilon_0}\int_{x=0}^{x=L}\frac{\lambda dx}{r^2}cos\theta; \left(r^2 = z^2 + x^2; cos\theta = \frac{z}{r}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\lambda z\int_0^L \frac{1}{(z^2+x^2)^{\frac{3}{2}}}dx = \frac{1}{4\pi\varepsilon_0}\lambda z\left[\frac{1}{z^2}\frac{x}{\sqrt{z^2+x^2}}\right]_{x=0}^{x=L} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z}\frac{L}{\sqrt{z^2+L^2}}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda dx}{r^2}sin\theta = -\frac{1}{4\pi\varepsilon_0}\lambda\int_0^L \frac{xdx}{(z^2+x^2)^{\frac{3}{2}}} = -\frac{1}{4\pi\varepsilon_0}\lambda\left[-\frac{1}{\sqrt{z^2+x^2}}\right]_{x=0}^{x=L}$$

$$= -\frac{1}{4\pi\varepsilon_0}\lambda\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+L^2}}\right]$$

Net electric field $E = E_x + E_z$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{z}\left[\left(\frac{z}{\sqrt{z^2+L^2}} - 1\right)\hat{x} + \left(\frac{L}{\sqrt{z^2+L^2}}\right)\hat{z}\right]$$

For $z >> L$ *and* $q = \lambda L; E \to \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{z^2}\hat{z}$