In order to make a split while bowling, it is determined that one of the pins needs to be hit and forced to move at a 70deg angle from the direction of the incoming ball. The ball's mass is 5times the pins mass. It is moving at 12 m/s when the ball strikes the pin. It can be assumed the collision is perfectly elastic, find the final speed of the ball and the pin and the ancgle at which the ball ends up moving.

Solution

$V = 12\frac{m}{s}, \alpha = 70°$, M=5m

the final speed of the ball - V_2

the final speed of the pin - V_1

the angle at which the ball ends up moving – β

According to the law of conservation of momentum for projections on the axes

$$mV_1 * sin\alpha = 5mV_2 * sin\beta \rightarrow V_1 sin\alpha = 5V_2 sin\beta.$$

$$5mV = 5mV_2 cos\beta + mV_1 * cos\alpha \rightarrow 5V = 5V_2 cos\beta + V_1 cos\alpha$$

According to the law of conservation of energy when the collision is perfectly elastic

$$\frac{5mV^2}{2} = \frac{5mV_2^2}{2} + \frac{mV_1^2}{2} \rightarrow 5V^2 = 5V_2^2 + V_1^2.$$

$$\begin{cases} V_1 sin\alpha = 5V_2 sin\beta \\ 5V - V_1 cos\alpha = 5V_2 sin\beta \end{cases} \rightarrow (V_1 sin\alpha)^2 + (5V - V_1 cos\alpha)^2 = 25V_2^2.$$

$$V_1^2 \sin^2\alpha + 25V^2 + V_1^2 \cos^2\alpha - 2 * 5VV_1 cos\alpha = 25V_2^2. \rightarrow$$

$$\begin{cases} V_1^2 + 25V^2 - 10VV_1 cos\alpha = 25V_2^2 \\ 5V^2 = 5V_2^2 + V_1^2. \end{cases} \rightarrow V_1^2 + 25V^2 - 10VV_1 cos\alpha = 25V^2 - 5V_1^2$$

$$6V_1^2 - 10VV_1 cos\alpha = 0 \rightarrow 6V_1 - 10V cos\alpha = 0 \rightarrow V_1 = \frac{10V cos\alpha}{6} = \frac{5}{3} V cos\alpha = 6.84\frac{m}{s}$$

$$V_2^2 = \frac{1}{5}(5 * 12^2 - 6.84^2) = 134.64\frac{m^2}{s^2}$$

$$V_2 = 11.6\frac{m}{s}$$

$$sin\beta = \frac{V_1 sin\alpha}{5V_2} = \frac{6.84 * 0.94}{5 * 11.6} = 0.11$$

$$\beta = 6.36°$$

Answer: $V_2 = 11.6\frac{m}{s}$, $V_1 = 6.84\frac{m}{s}$, $\beta = 6.36°$