Four identical particles each have a charge 0.4microC and a mass 0.04kg. They are released from rest at the vertices of a square of side .36m. How fast is each charge moving when their distances from the center of the square doubles? The Coulomb constant is 8.98755 X 10^9 N*m^2/C^2. Answer in units of m/s

Solution

Since the particles will be in motion, the force is variant with respect to time, and consequently determining the velocity function of the particles will involve setting up and solving a differential equation. In this instance, because the situation is so symmetric, there is a particularly elegant (and calculus free!) solution utilizing the principle of conservation of energy.

It is possible to show that the work necessary to move two charges q1 and q2 from infinity to a distance d from one another is k*q1*q2/d, where k is the Coulomb constant. By the principle of superposition, if more than two charges are present, we just sum up the k*q1*q2/d terms for each possible pair of charges. Since there are four charges here, there are C(4, 2) = 6 pairs. If you number off vertices around the square, these pairs would be (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4). Since (1,3) and (2,4) are diagonals, the distance between those vertices is (0.36√2 m), but all other distances are 0.36m. The symmetry of the situation (symmetric arrangement, same mass, same charge) makes the total potential energy of the system easy to determine:

$$\frac{4k(0.4\ \mu C)^2}{0.36m} + \frac{2k(0.4\ \mu C)^2}{0.36\sqrt{2}m} = 21{,}615 * 10^{-3}\ J.$$

Since the particles are released from rest, their initial kinetic energy is zero. Now we need to determine the distances between the charges once their mutual distance to the center has doubled. Since the distance to center (i.e. half the diagonal) has doubled in length, the diagonal doubles in length to 0.72√2. Since the side length and diagonal are in a ratio of 1:√2, the new side lengths are 0.72√2/√2 = 0.72. Alternatively, you could just observe that all relative lengths must double because of similar triangles. In any case, the new PE of the system is:

$$\frac{4k(0.4\ \mu C)^2}{0.72\ m} + \frac{2k(0.4\ \mu C)^2}{0.72 * \sqrt{2}m} = 10{,}803 * 10^{-3}\ J.$$

Per conservation of energy,

$$\Delta K = -\Delta U = -(10{,}803 * 10^{-3}J - 21{,}615 * 10^{-3}J) = 10{,}812 * 10^{-3}\ J.$$

Here is where symmetry is very handy: there is no way to distinguish one particle from another, because apart from direction, all particles carry the same charge, have the same mass, and experience the same force. Hence it must be the case that all four particles each carry the same amount of this kinetic energy:

$$\frac{10{,}812}{4} * 10^{-3} = 2{,}703 * 10^{-3}\ J.$$

KE = 1/2mv². Initial KE was zero, so the final KE of each particle is equal to $= 2{,}703 * 10^{-3}\ J.$

$$2{,}703 * 10^{-3}J = (1/2)(0.04kg)v^2$$

$$2 * (2{,}703 * 10^{-3})/(0.04) = v^2 = 0{,}13515\ m^2/s^2$$

$$v = \sqrt{(0{,}13515\frac{m^2}{s^2})} = 0.37\ m/s.$$