How to find final height in a work, kinetic energy problem with only the initial velocity, initial height and an angle? The object goes off a ramp with some projectile motion.

Answer

The increase of the height will last, until $v_y = 0$.

$$0 = v_0 \sin(\alpha) - g t_h,$$

Time to reach the maximum height:

$$t_h = \frac{v_0 \sin(\alpha)}{g}.$$

From the vertical displacement the maximum height of projectile:

$$h = v_0 t_h \sin(\alpha) - \frac{1}{2} g t_h^2,$$

so

$$h = \frac{v_0^2 \sin^2 \alpha}{2g}.$$