Question 15512

From one side, one knows that

$$v_x = v_0 \cos\varphi, S_x = T v_0 \cos\varphi = 200\,m \quad (1)$$

For vertical motion,

$$S_y = v_0 t \sin\varphi - \frac{g t^2}{2} \quad (2)$$

The ball reaches the maximum altitude, when $\frac{d S_y}{dt} = 0, \Rightarrow t_{1/2} = \frac{v_0 \sin\varphi}{g}$ - this gives the half of the time of the flight, so the full time of the flight is

$$T = \frac{2 v_0 \sin\varphi}{g} \quad (3)$$

Now using (1) and (3), obtain: $\sin 2\varphi = \frac{S \cdot g}{v_0^2}$, which gives $\varphi_1 \approx 16.51$ for $\varphi \in [0, \pi/2]$. The second angle, is obviously $\varphi_2 = \pi - 2\varphi_1 \approx 146.98$ (this will give the motion to the left, if for $\varphi_1 \approx 16.51$ the motion was to the right). The time of the flight and maximum altitude will be equal for both angles. Hence,

using (3), $T = \frac{2 v_0 \sin\varphi}{g} \approx 3.48\,s$, and

using (2), $h_{max} = S_y|_{t=t_{1/2}} = [v_0 t \sin\varphi - \frac{g t^2}{2}]|_{t=t_{1/2}} = \frac{v_0^2 \sin^2\varphi}{2g} \approx 14.82\,m$.