Applicability of the strict mass–energy equivalence formula, $E = mc^2$

As is noted above, two different definitions of mass have been used in special relativity, and also two different definitions of energy. The simple equation $E = mc^2$ is not generally applicable to all these types of mass and energy, except in the special case that the total additive momentum is zero for the system under consideration. In such a case, which is always guaranteed when observing the system from either its center of mass frame or its center of momentum frame, $E = mc^2$ is always true for any type of mass and energy that are chosen. Thus, for example, in the center of mass frame, the total energy of an object or system is equal to its rest mass times c^2, a useful equality. This is the relationship used for the container of gas in the previous example. It is *not* true in other reference frames where the center of mass is in motion. In these systems or for such an object, its total energy will depend on both its rest (or invariant) mass, and also its (total) momentum.

In inertial reference frames other than the rest frame or center of mass frame, the equation $E = mc^2$ remains true if the energy is the relativistic energy *and* the mass the relativistic mass. It is also correct if the energy is the rest or invariant energy (also the minimum energy), *and* the mass is the rest mass, or the invariant mass. However, connection of the**total or relativistic energy** (E_r) with the **rest or invariant mass** (m_0) requires consideration of the system total momentum, in systems and reference frames where the total momentum has a non-zero value. The formula then required to connect the two different kinds of mass and energy, is the extended version of Einstein's equation, called therelativistic energy–momentum relationship:

$$E_r^2 - |\vec{p}|^2 c^2 = m_0^2 c^4$$
$$E_r^2 - (pc)^2 = (m_0 c^2)^2$$

or

$$E_r = \sqrt{(m_0 c^2)^2 + (pc)^2}$$

Here the $(pc)^2$ term represents the square of the Euclidean norm (total vector length) of the various momentum vectors in the system, which reduces to the square of the simple momentum magnitude, if only a single particle is considered. This equation reduces to $E = mc^2$ when the momentum term is zero. For photons where $m_0 = 0$, the equation reduces to $E_r = pc$.