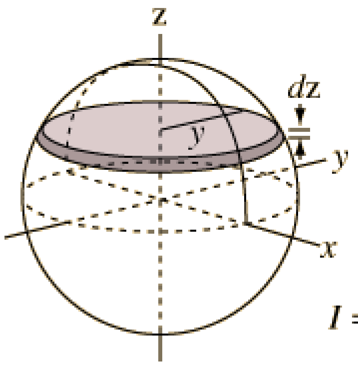
explain moment of interia of a solid sphere and solid cylinder?

Solid ball of radius R and mass M:

$I = 2MR^2/5$

The expression for the moment of inertia of a sphere can be developed by summing the moments of infintesmally thin disks about the z axis. The moment of inertia of a thin disk is



$$dI = \frac{1}{2}y^2 dm = \frac{1}{2}y^2 \rho dV = \frac{1}{2}y^2 \rho\pi y^2 dz$$

and the integral becomes

$$I = \frac{1}{2}\rho\pi\int_{-R}^{R} y^4 dz = \frac{1}{2}\rho\pi\int_{-R}^{R}(R^2 - z^2)^2 dz = \frac{8}{15}\rho\pi R^5$$

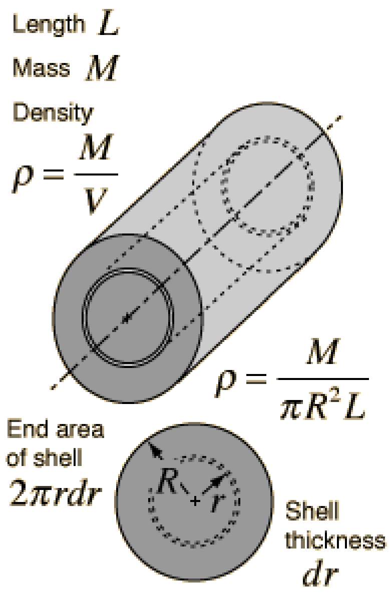
Radius = R
Mass = M
Density $= \rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$

Substituting the density expression gives

$$I = \frac{8}{15}\left[\frac{M}{\frac{4}{3}\pi R^3}\right]\pi R^5 = \frac{2}{5}MR^2$$

Solid cylinder of radius R, height H and mass M:

$I = MR^2/2$



The expression for the moment of inertia of a solid cylinder can be built up from the moment of inertia of thin cylindrical shells. Using the general definition for moment of inertia:

$$I = \int_0^M r^2 dm$$

The mass element can be expressed in terms of an infinitesmal radial thickness dr by

$$dm = \rho dV = \rho L 2\pi r dr$$

Substituting gives a polynomial form integral:

$$I = 2\pi\rho L \int_0^R r^3 dr = 2\pi\rho L \frac{R^4}{4}$$

$$I = 2\pi \left[\frac{M}{\pi R^2 L} \right] L \frac{R^4}{4} = \frac{1}{2} M R^2$$