An indoor track is to be designed such that each end is a banked semi-circle with a radius of 24 m. What should the banking angle be for a person running at speed v = 6.0 m/s?

Solution:

As opposed to a car riding along a flat circle, inclined edges add an additional force that keeps the car in its path and prevents it from being "dragged into" or "pushed out of" the circle. This force is the horizontal component of the car's normal force. In the absence of friction, the normal force is the only one acting on the car in the direction of the center of the circle. Therefore, as per Newton's second law, we can set the horizontal component of the normal force equal to mass multiplied by centripetal acceleration:

$$N \sin\theta = \frac{mv^2}{r}$$

Because there is no motion in the vertical direction, the sum of all vertical forces acting on the system must be zero. Therefore we can set the vertical component of the car's normal force equal to its weight:

$$N \cos\theta = mg$$

Solving the above equation for the normal force and substituting this value into our previous equation, we get:

$$\frac{mv^2}{r} = mg \tan\theta$$

which is equivalent to:

$$\frac{v^2}{r} = g \tan\theta$$

$$\tan\theta = \frac{v^2}{rg} = \frac{6^2}{24 \cdot 9.8} = 0{,}15306 \gg \theta = 8.7^\circ$$