Answer on Question #11567-Physics-Molecular Physics-Thermodynamics

A certain fluid at 10 bar is contained in a cylinder behind a piston, the initial volume being 0.05 m^3. Calculate the work done by the fluid when it expands reversibly:

i. at constant pressure to a final volume of 0.2 m^3

ii. according to a linear law to a final volume of 0.2 m^3 and a final pressure of 2 bar

iii. according to a law pV = constant to a final volume of 0.1 m^3

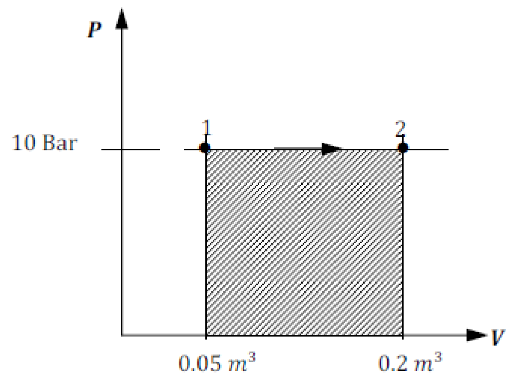
iv. according to a law pV^3 = constant to a final volume 0f 0.06 m^3

v. according to a law p=(a/v^2) - (b/v) to a final volume of 0.1 m^3 and a final pressure of 1 bar where A and B are constants.

Sketch all processes on a p-V diagram.

Solution

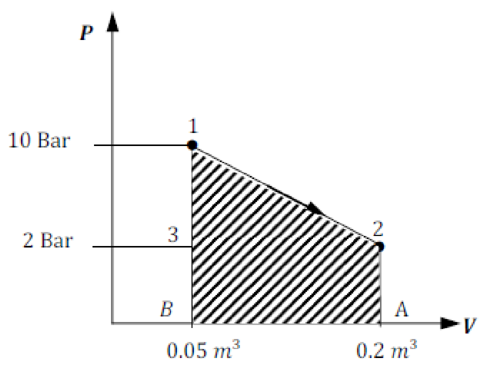
i.



For reversible non-flow process we have

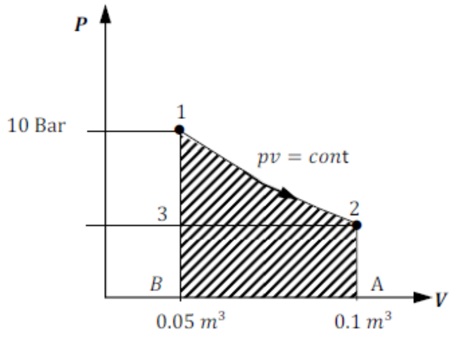
$$W = shaded\ area = \int_{V_1}^{V_2} pdV = p(V_2 - V_1) = 1000\ kPa \cdot (0.2\ - 0\ .05)m^3\ =\ 150\ kJ.$$

ii.



$$W = shaded\ area = \left(\frac{p_1 + p_2}{2}\right)|V_2 - V_1| = \left(\frac{10 + 2}{2}\right)10^5|0.2 - 0.05| = 90\ kJ.$$

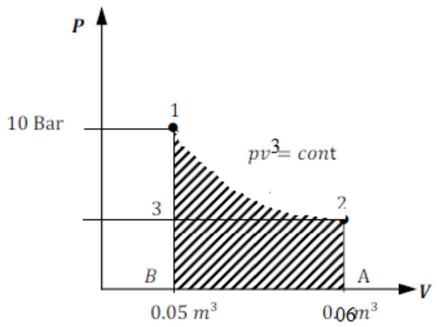
iii.



$$W = shaded\ area = \int_1^2 pdV = \int_1^2 \frac{p_1 V_1}{V} dV.$$

$$W = p_1 V_1 \ln \frac{V_2}{V_1} = 1000 \cdot 0.05 \cdot \ln\left(\frac{0.1}{0.05}\right) = 34.7\ kJ.$$

iv.



$$W = \frac{p_2 V_2 - p_1 V_1}{(1-3)}.$$

Where $p_2 = p_1 \left(\frac{V_1}{V_2}\right)^3 = 10 \left(\frac{0.05}{0.06}\right)^3 = 5.787\ bar.$

$$W = \frac{5.787 \cdot 0.06 - 10 \cdot 0.05}{-2} 10^5 = 7640\ J.$$

v.

$$W = \int_1^2 \left(\left(\frac{\mathrm{a}}{\mathrm{v}^2}\right) - \left(\frac{\mathrm{b}}{\mathrm{v}}\right)\right) dV = a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) + b \ln \frac{V_1}{V_2}.$$

$$10bar = \left(\frac{\mathrm{a}}{(0.05\ \mathrm{m}^3)^2}\right) - \left(\frac{\mathrm{b}}{0.05\ \mathrm{m}^3}\right);\ 1bar = \left(\frac{\mathrm{a}}{(0.1\ \mathrm{m}^3)^2}\right) - \left(\frac{\mathrm{b}}{0.1\ \mathrm{m}^3}\right).$$

$$a = 0.04\ bar\ m^6; b = 0.3\ bar\ m^3.$$

Thus,

$$W = 0.04\ bar\ m^6 \left(\frac{1}{0.05\ \mathrm{m}^3} - \frac{1}{0.1\ \mathrm{m}^3}\right) + 0.3\ bar\ m^3 \ln \frac{0.05\ \mathrm{m}^3}{0.1\ \mathrm{m}^3} = 19.2\ kJ.$$