

The magnitude of displacement of a particle moving in a circle of radius 'a' with constant angular speed ω varies with time 't' as :

(A) $2 a \sin \omega t$;

(B) $2 a \sin \omega t/2$;

(C) $2 a \cos \omega t$;

(D) $2 a \cos \omega t/2$;

Please do specify the method by which u got ur answer...

Particle position:

$$\vec{R} = a \cos(\omega t) \vec{i} + a \sin(\omega t) \vec{j}$$

At $t=0$:

$$\vec{R}_0 = a \vec{i}$$

Displacement:

$$\vec{d} = \vec{R} - \vec{R}_0 = a \cos(\omega t) \vec{i} + a \sin(\omega t) \vec{j} - a \vec{i} = a(\cos(\omega t) - 1) \vec{i} + a \sin(\omega t) \vec{j}$$

$$d = \sqrt{(a(\cos(\omega t) - 1))^2 + (a \sin(\omega t))^2} = a\sqrt{2 - 2 \cos(\omega t)} = a\sqrt{2(1 - \cos(\omega t))}$$

$$1 - \cos(\omega t) = 2(\sin(\omega t/2))^2$$

$$d = a\sqrt{2 * 2(\sin(\omega t/2))^2} = 2a \sin(\omega t/2)$$

Answer $d = 2a \sin(\omega t/2)$ (B)