

Question #9886

Show that $n^4 - 1$ is divisible by 8, if n is an odd positive integer.

Solution. Write $n^4 - 1 = (n^2 + 1)(n - 1)(n + 1)$. If $n = 2k + 1$, then $n - 1$, $n + 1$ are divisible by 2, hence $(n - 1)(n + 1)$ is divisible by 4, $n^2 + 1 = 4k^2 + 4k + 2$ is also divisible by 2, thus we get that $(n^2 + 1)(n - 1)(n + 1)$ is divisible by 8.