Let find eigen values. For that we need the next formula:

$$\det(A-\lambda I)=0$$

Where I is the identity matrix

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2(3-\lambda)-4+2+2(3-\lambda)-2(2-\lambda)-2(2-\lambda)$$
$$= -\lambda^3+7\lambda^2-14\lambda+8$$

$$-\lambda^3+7\lambda^2-14\lambda+8=0$$

$$\lambda^3-7\lambda^2+14\lambda-8=0$$

$$(\lambda-1)(\lambda-2)(\lambda-4)=0$$

$$\left[\begin{array}{l} \lambda-1=0 \\ \lambda-2=0 \\ \lambda-4=0 \end{array}\right. => \left[\begin{array}{l} \lambda_1=1 \\ \lambda_2=2 \\ \lambda_3=4 \end{array}\right.$$

Now we will find the eigen vectors corresponding to each eigen value

1. For $\lambda_1=1$

$$(A-\lambda_1)\alpha=0$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -2 & 1 & 1 \end{bmatrix} * \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = 0$$

   Having the system of three equations

$$\begin{cases} \alpha_1+\alpha_2+\alpha_3=0 \\ 2\alpha_1+2\alpha_2+2\alpha_3=0 \\ -2\alpha_1+\alpha_2+\alpha_3=0 \end{cases}$$

   Solve the system by the Gauss method

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

   Having the next equations

$$\begin{cases} \alpha_1+\alpha_2+\alpha_3=0 \\ \alpha_2+\alpha_3=0 \end{cases}$$

$$\begin{cases} \alpha_1+\alpha_2=-\alpha_3 \\ \alpha_2=-\alpha_3 \end{cases}$$

   Let $\alpha_3=t$, than $\alpha_2=-t$ and $\alpha_1=0$. That means that our eigen vector has the following form:

$$\alpha=\begin{pmatrix} 0 \\ -t \\ t \end{pmatrix} \text{ or } \alpha=\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

2. For $\lambda_2=2$

   Similar to the previous

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ -2 & 1 & 0 \end{bmatrix} * \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = 0$$

$$\begin{pmatrix} 0 & 1 & 1 & | & 0 \\ 2 & 1 & 2 & | & 0 \\ -2 & 1 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 1 & | & 0 \\ 2 & 1 & 2 & | & 0 \\ 0 & 2 & 2 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 2 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1/2 & 1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\begin{cases} \beta_1 + \frac{1}{2}\beta_2 + \beta_3 = 0 \\ \beta_2 + \beta_3 = 0 \end{cases}$$

$$\begin{cases} \beta_1 + \frac{1}{2}\beta_2 = -\beta_3 \\ \beta_2 = -\beta_3 \end{cases}$$

Let $\beta_3 = t => \beta_2 = -t => \beta_1 = -\frac{1}{2}t => \beta = \begin{pmatrix} -\frac{1}{2}t \\ -t \\ t \end{pmatrix} \; or \; \beta = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

3. For $\lambda_3 = 4$
   Similar to the previous

$$\begin{bmatrix} -2 & 1 & 1 \\ 2 & -1 & 2 \\ -2 & 1 & -2 \end{bmatrix} * \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = 0$$

$$\begin{pmatrix} -2 & 1 & 1 & | & 0 \\ 2 & -1 & 2 & | & 0 \\ -2 & 1 & -2 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 3 & | & 0 \\ 2 & -1 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & 1 & | & 0 \\ 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\begin{cases} \gamma_1 - \frac{1}{2}\gamma_2 + \gamma_3 = 0 \\ \gamma_3 = 0 \end{cases}$$

$$\begin{cases} \gamma_1 = \frac{1}{2}\gamma_2 \\ \gamma_3 = 0 \end{cases}$$

Let $\gamma_2 = t => \; \gamma_1 = \frac{1}{2}t => \; \gamma = \begin{pmatrix} \frac{1}{2}t \\ t \\ 0 \end{pmatrix} \; or \; \gamma = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

Answer: 1) $\lambda_1 = 1; \; \alpha = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

2) $\lambda_2 = 2; \; \beta = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

3) $\lambda_3 = 4; \; \gamma = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$