Question #9664 Calculate $\lim_{x\to 0} \dfrac{(\log\cos x)}{\log\cos 3x}$.

Solution. Use $\log(1+x) = x + o(x), x \to \infty$. Hence $\log(\cos x) = \cos x - 1 + o(\cos x - 1) = 1/2x^2 + o(x^2)$. Thus, $\log(\cos 3x) = 1/2 \cdot 9x^2 + o(x^2)$. So, $\lim_{x\to 0} \dfrac{(\log\cos x)}{\log\cos 3x} = \lim_{x\to 0} \dfrac{1/2x^2 + o(x^2)}{1/2 \cdot 9x^2 + o(x^2)} = \dfrac{1}{9}$.

Answer. $\dfrac{1}{9}$.

1