

In [statistics](#), the **mode** is the value that occurs most frequently in a [data set](#) or a [probability distribution](#).

the mode is a way of [capturing important information](#) about a [random variable](#) or a [population](#) in a single quantity. The mode is in general different from the mean and median, and may be very different for strongly skewed distributions.

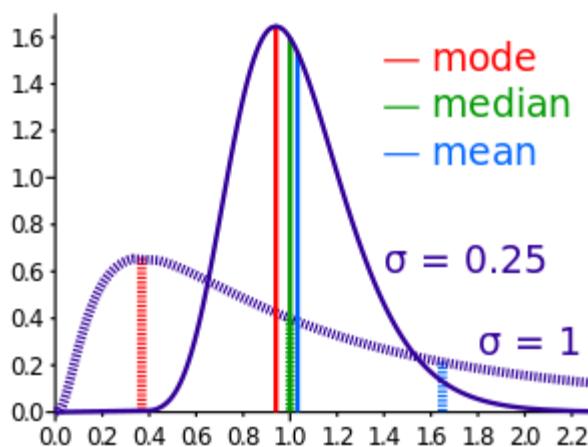
## Mode of a sample

The mode of a sample is the element that occurs most often in the collection. For example, the mode of the sample [1, 3, 6, 6, 6, 6, 7, 7, 12, 12, 17] is 6. Given the list of data [1, 1, 2, 4, 4] the mode is not unique - the dataset may be said to be [bimodal](#), while a set with more than two modes may be described as [multimodal](#).

For a sample from a continuous distribution, such as [0.935..., 1.211..., 2.430..., 3.668..., 3.874...], the concept is unusable in its raw form, since each value will occur precisely once. The usual practice is to discretize the data by assigning frequency values to [intervals](#) of equal distance, as for making a [histogram](#), effectively replacing the values by the midpoints of the intervals they are assigned to. The mode is then the value where the histogram reaches its peak. For small or middle-sized samples the outcome of this procedure is sensitive to the choice of interval width if chosen too narrow or too wide; typically one should have a sizable fraction of the data concentrated in a relatively small number of intervals (5 to 10), while the fraction of the data falling outside these intervals is also sizable. An alternate approach is [kernel density estimation](#), which essentially blurs point samples to produce a continuous estimate of the probability density function which can provide an estimate of the mode.

### Example for a skewed distribution

An example of a [skewed](#) distribution is [personal wealth](#): Few people are very rich, but among those some are extremely rich. However, many are rather poor.



Comparison of [mean](#), [median](#) and [mode](#) of two [log-normal distributions](#) with different [skewness](#).

A well-known class of distributions that can be arbitrarily skewed is given by the [log-normal distribution](#). It is obtained by transforming a random variable  $X$  having a normal distribution into random variable  $Y = e^X$ . Then the logarithm of random variable  $Y$  is normally distributed, hence the name.

Taking the mean  $\mu$  of  $X$  to be 0, the median of  $Y$  will be 1, independent of the [standard deviation](#)  $\sigma$  of  $X$ . This is so because  $X$  has a symmetric distribution, so its median is also 0. The transformation from  $X$  to  $Y$  is monotonic, and so we find the median  $e^0 = 1$  for  $Y$ .

When  $X$  has standard deviation  $\sigma = 0.25$ , the distribution of  $Y$  is weakly skewed. Using formulas for the [log-normal distribution](#), we find:

$$\begin{array}{llll} \text{mean} & = e^{\mu+\sigma^2/2} & = e^{0+0.25^2/2} & \approx 1.032 \\ \text{mode} & = e^{\mu-\sigma^2} & = e^{0-0.25^2} & \approx 0.939 \\ \text{median} & = e^\mu & = e^0 & = 1 \end{array}$$

Indeed, the median is about one third on the way from mean to mode.

When  $X$  has a larger standard deviation,  $\sigma = 1$ , the distribution of  $Y$  is strongly skewed. Now

$$\begin{array}{llll} \text{mean} & = e^{\mu+\sigma^2/2} & = e^{0+1^2/2} & \approx 1.649 \\ \text{mode} & = e^{\mu-\sigma^2} & = e^{0-1^2} & \approx 0.368 \\ \text{median} & = e^\mu & = e^0 & = 1 \end{array}$$

Here, [Pearson's rule of thumb](#) fails.