

Question #9394

Simplify: $\sin(1 + \operatorname{ctg} 2x)$

Solution

$$\begin{aligned}\sin(1 + \operatorname{ctg} 2x) &= \sin 1 \cos(\operatorname{ctg} 2x) + \cos 1 \sin(\operatorname{ctg} 2x) = \sin 1 \cos\left(\frac{\cos 2x}{\sin 2x}\right) + \cos 1 \sin\left(\frac{\cos 2x}{\sin 2x}\right) = \\ &= \sin 1 \cos \frac{1}{2}(\operatorname{ctg} x - \operatorname{tg} x) + \cos 1 \sin \frac{1}{2}(\operatorname{ctg} x - \operatorname{tg} x) = \sin 1 \cos\left(\frac{\operatorname{ctg} x}{2} - \frac{\operatorname{tg} x}{2}\right) + \cos 1 \sin\left(\frac{\operatorname{ctg} x}{2} - \frac{\operatorname{tg} x}{2}\right) = \\ &= \sin 1 \left(\cos \frac{\operatorname{ctg} x}{2} \cos \frac{\operatorname{tg} x}{2} + \sin \frac{\operatorname{ctg} x}{2} \sin \frac{\operatorname{tg} x}{2}\right) + \cos 1 \left(\sin \frac{\operatorname{ctg} x}{2} \cos \frac{\operatorname{tg} x}{2} - \cos \frac{\operatorname{ctg} x}{2} \sin \frac{\operatorname{tg} x}{2}\right) = \\ &= \sin 1 \cos \frac{\operatorname{ctg} x}{2} \cos \frac{\operatorname{tg} x}{2} + \sin 1 \sin \frac{\operatorname{ctg} x}{2} \sin \frac{\operatorname{tg} x}{2} + \cos 1 \sin \frac{\operatorname{ctg} x}{2} \cos \frac{\operatorname{tg} x}{2} - \cos 1 \cos \frac{\operatorname{ctg} x}{2} \sin \frac{\operatorname{tg} x}{2} = \\ &= \cos \frac{\operatorname{tg} x}{2} \sin\left(1 + \frac{\operatorname{ctg} x}{2}\right) + \sin \frac{\operatorname{tg} x}{2} \cos\left(\frac{\operatorname{ctg} x}{2} - 1\right)\end{aligned}$$