

Solve the system:

$$1/r + 3/s - 2/t = 1$$

$$2/r + 3/s - 4/t = 1$$

$$1/r - 6/s - 6/t = 0$$

Solution

We carry out a change of variables

$$\frac{1}{r} = x, \frac{1}{s} = y, \frac{1}{t} = z \gg \begin{cases} x + 3y - 2z = 1 \\ 2x + 3y - 4z = 1 \\ x - 6y - 6z = 0 \end{cases} \gg \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 2 & 3 & -4 & 1 \\ 1 & -6 & -6 & 0 \end{array} \right) \gg$$

$$\gg \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & -3 & 0 & -1 \\ 0 & 9 & 4 & 1 \end{array} \right) \gg \begin{cases} x + 3y - 2z = 1 \\ -3y = -1 \\ 9y + 4z = 1 \end{cases} \gg$$

$$\gg y = \frac{1}{3}, 9\frac{1}{3} + 4z = 1 \gg z = -\frac{1}{2}, x + 3\frac{1}{3} - 2\left(-\frac{1}{2}\right) = 1 \gg x = -1$$

$$r = \frac{1}{x} = -1, s = \frac{1}{y} = 3, t = \frac{1}{z} = -2$$