The task:

25.) Use the Rational Roots Theorem to solve the equation for the rational roots.

$$4y^5 + 8y^4 - 29y^3 - 42y^2 + 45y + 54 = 0$$

Solution:

Let's use Horner's scheme, will be sought roots of the divisors of the free term, 54. That is $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm27, \pm54$

	4	8	-29	-42	45	54
1	4	12	-17	-59	-14	$40 \neq 0$
-1	4	4	-33	-9	54	0

That means that $4y^5 + 8y^4 - 29y^3 - 42y^2 + 45y + 54 = (y+1)(4y^4 + 4y^3 - 33y^2 - 9y + 54)$

Analogically

	4	4	-33	-9	54
1	4	8	-25	-34	≠0
-1	4	0	-33	24	≠0
2	4	12	-9	-27	0

So, $4y^5 + 8y^4 - 29y^3 - 42y^2 + 45y + 54 = (y+1)(y-2)(4y^3 + 12y^2 - 9y - 27)$

Consider the next polynomial

$$4y^3 + 12y^2 - 9y - 27 = 4y^2(y+3) - 9(y+3) = (y+3)(4y^2 - 9)$$
$$= (y+3)(2y-3)(2y+3)$$

So, $4y^5 + 8y^4 - 29y^3 - 42y^2 + 45y + 54 = (y+1)(y-2)(y+3)(2y-3)(2y+3) = 0$

Expression is equal to 0 if at least one of the factors is 0

$$\begin{bmatrix} y = -1 \\ y = 2 \\ y = -3 \\ y = \frac{3}{2} \\ y = -\frac{3}{2} \end{bmatrix}$$

Answer: -1; 2; -3; ±(3/2)