Answer to Question #91542 – Math – Analytic Geometry

Question

Obtain the equation of the conic, a focus of which lies at (2,1), the directrix of which is x+y=0 and which passes through (1,4). Also identify the conic.

Solution

We need to find the distance between (2,1) and (1,4)

Using distance formula:

$$\sqrt{(1-2)^2+(4-1)^2}$$

$$\sqrt{(1+9)}$$

$$\sqrt{10}$$

Distance of the point (1,4) from the directrix x+y = 0 is

$$\frac{(1+4)}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

Ratio of distance is $\frac{\sqrt{10}}{5/\sqrt{2}}$

This ration is less than 1, so this is an ellipse.

Its equation is obtained from ratio of the distance of a point on ellipse say (x,y) from focus (2,1) and its distance from the directrix x+y=0 is being 2 / √5.

The latter is x+y / √2.

Thus, the equation is

$$\frac{(x-2)^2+(y-1)^2}{\left(\frac{x+y}{\sqrt{2}}\right)^2} = \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5}$$

$5\,(x^2 - 4x + 4 + y^2 - 2y + 1) = 2\,(x^2 + 2xy + y^2)$

$5x^2 - 20x + 20 + 5y^2 - 10y + 5 = 2x^2 + 4xy + 2y^2$

$3x^2 - 4xy - 20x + 3y^2 - 10y + 25 = 0$

This conic is Ellipse.