Question: Choose the correct answer.

Which of the following statement is true for sequence $\{a_n = (-1)^{n-1}\}$?

    a. The sequence is bounded
    b. The sequence is increasing
    c. The sequence is decreasing
    d. The sequence is neither increasing nor decreasing

Solution

The option (a) is Correct.

Explanation

The nth term of the sequence is $a_n = (-1)^{n-1}$

Hence the sequence is

$$a_1 = (-1)^{1-1} = 1, \qquad a_2 = (-1)^{2-1} = -1, \qquad a_3 = (-1)^{3-1} = 1, \qquad a_4 = (-1)^{4-1} = -1 \qquad \ldots$$

We can see that for all values of $n \in \mathbb{N}$, there exist two numbers 1 and -1 such that $a_n \leq 1$ for all even n, and $a_n \geq -1$ for all odd n.

The two number 1 and -1 are called lower and upper bound.

Hence the series is bounded above and bounded below.

Bounded Sequences of Real Numbers

A sequence a_n; $n = 1, 2, 3, \ldots$ of real numbers is said to be **Bounded Above** if there exists a real number $M \in \mathbb{R}$ such that $a_n \leq M$ for every $n \in \mathbb{N}$. And if for $m \in \mathbb{R}$ such that $m \leq a_n$ for every $n \in \mathbb{N}$, the sequence is called **Bounded Below**.

Taking n along horizontal and $(-1)^{n-1}$ along vertical we get the graph as shown by the dots below.