

Question #9107

Given $z_1 = 3(\cos 14^\circ + i \sin 14^\circ)$ and $z_2 = 2(\cos 121^\circ + i \sin 121^\circ)$, find the product $z_1 \cdot z_2$

Solution

$$z_1 = r_1(\cos \alpha_1 + i \sin \alpha_1)$$

$$z_2 = r_2(\cos \alpha_2 + i \sin \alpha_2)$$

$$\begin{aligned} z_1 z_2 &= r_1 r_2 (\cos \alpha_1 \cos \alpha_2 + i \cos \alpha_1 \sin \alpha_2 + i \sin \alpha_1 \cos \alpha_2 + i^2 \sin \alpha_1 \sin \alpha_2) = \\ &= r_1 r_2 ((\cos \alpha_1 \cos \alpha_2 - \sin \alpha_1 \sin \alpha_2) + i(\sin \alpha_1 \cos \alpha_2 + \cos \alpha_1 \sin \alpha_2)) = \\ &= r_1 r_2 (\cos(\alpha_1 + \alpha_2) + i \sin(\alpha_1 + \alpha_2)) \end{aligned}$$

$$z_1 \cdot z_2 = 2 \cdot 3 (\cos(14^\circ + 121^\circ) + i \sin(14^\circ + 121^\circ)) = 6(\cos(135^\circ) + i \sin(135^\circ))$$

Answer: $z_1 \cdot z_2 = 6(\cos(135^\circ) + i \sin(135^\circ))$