

Answer to Question #91049 – Math – Algebra

Question

Determine the value of a and b in the following
 (x^2+x-2) is a factor of $3x^3-x^2+ax+b$

Solution

Given,

$x^2 + x - 2$ is a factor of

$$f(x) = 3x^3 - x^2 + ax + b$$

Therefore, factors of $x^2 + x - 2$ will be factors of $3x^3 - x^2 + ax + b$ also.

$$x^2 + x - 2 = x^2 + 2x - x + 2$$

$$x^2 + 2x - x + 2 = x \times (x + 2) - 1 \times (x + 2)$$

$$x \times (x + 2) - 1 \times (x + 2) = (x - 1)(x + 2)$$

Therefore,

$$(x - 1)(x + 2) = 0$$

$$x=1$$

Or

$$x=-2$$

Since these are factors of $f(x)$ also therefore $f(1)=0$ and $f(-2)=0$

$$f(1) = 3(1)^3 - (1)^2 + a(1) + b=0$$

$$3-1+a+b=0$$

$$2+a+b=0$$

$$a+b=-2\text{.....(1)}$$

$$f(-2) = 3(-2)^3 - (-2)^2 + a(-2) + b=0$$

$$-24-4-2a+b=0$$

$$-28-2a+b=0$$

$$-2a+b=28\text{.....(2)}$$

On subtracting equation (2) from equation (1)

$$a+b-(-2a+b)=-2-28$$

$$3a=-30$$

$$a=\frac{-30}{3} = -10$$

on substituting this value of a in equation (1)

$$-10 + b = -2$$

$$b = 8$$

Therefore,

$$a=-10 \text{ and } b=8.$$

Answer: a=-10 and b=8.