Answer to Question #90824 – Math – Trigonometry

Question

a^3sin(B-C) +b^3sin(C-A) +c^3sin(A-B) =0

Solution

Take L.H.S,

$$= a^3 \sin(B - C) + b^3 \sin(C - A) + c^3 \sin(A - B)$$

Using Sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k \text{ [where } k \text{ is a constant]...(i)}$$

Using Cosine rule,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \cos C = \frac{b^2 + a^2 - c^2}{2ba} \text{...(ii)}$$

Consider,

$$a^3 \sin(B - C) = a^3 \left[\sin B \cos C - \cos B \sin C\right]$$

Plug values from (i) and (ii):

$$= a^3 \left[kb \times \frac{b^2 + a^2 - c^2}{2ba} - \frac{a^2 + c^2 - b^2}{2ac} \times kc \right]$$

$$= a^3 \left[k \times \frac{b^2 + a^2 - c^2}{2a} - \frac{a^2 + c^2 - b^2}{2a} \times k \right]$$

$$= ka^3 \left[\frac{b^2 + a^2 - c^2}{2a} - \frac{a^2 + c^2 - b^2}{2a} \right]$$

$$= ka^3 \left[\frac{2b^2 - 2c^2}{2a} \right]$$

$$= ka^2 \left[b^2 - c^2 \right]$$

Similarly, using values from (i) and (ii) to get,

$$b^3 \sin(C-A) = kb^2\left[c^2 - a^2\right]$$

$$c^3 \sin(A-B) = kc^2\left[a^2 - b^2\right]$$

Now putting all these values in L.H.S., we get,

$$= ka^2\left[b^2 - c^2\right] + kb^2\left[c^2 - a^2\right] + kc^2\left[a^2 - b^2\right]$$

$$= k\left[a^2b^2 - a^2c^2 + b^2c^2 - b^2a^2 + c^2a^2 - c^2b^2\right]$$

$$= k\left[a^2b^2 - a^2c^2 + b^2c^2 - a^2b^2 + a^2c^2 - b^2c^2\right]$$

$$= k\left[0\right]$$

$$= 0$$

$$= R.H.S$$

Hence, proved.