Answer on Question #90761 – Math – Combinatorics | Number Theory

Question

Secrets of Pascal triangle.

Solution

There are many identities which either involve Pascal's triangle for obtaining new identities based on combinations or relate various patterns of Pascal's triangle with other well-known series. Any criterion used for this selection cannot exhaust all combinations that can be derived.

First of all, the triangle is symmetrical (left and right sides are identical when an imaginary normal line is considered), the horizontal sums (per triangle's rows) always give powers of 2, and we obtain powers of 11 if we read all numbers per each row (e.g. $11^0=1$, $11^1=11$, $121=11^2$, $1331=11^3$).

For instance, any Fibonacci number can be obtained by summing-up the numbers obtained when going up in the triangle and then along (e.g. 1+6+10+4 =21, 1+5+6+1=13, 1+4+3=8, 1+3+1=5, 1+2=3, 1+1=2, 1,1).

Another one involves the number pi (π), namely it can be written as π= 3 + 2/3* (1/4C3 – 1/6C3 + 1/8C3 – 1/10C3 +…), where nCk =n!/[k!(n-k)!].

Pascal's triangle also allows reconstructing basic things/formulas like binomial expansion, binomial theorem or binomial distribution.

To close-up this shortlist of "secrets", it suffices to consider all diagonals of the triangle to can discover the following pattern: the 1st has only 1's, the 2nd gives the counting set N (positive integers), the 3rd diagonal gives the triangular numbers, and so forth.