Answer to Question #90156 – Math – Combinatorics | Number Theory

Question

From 24 to 12943 how many numbers can be made such that 1, 3, 5 do not appear 3 or more than 3 times?

Solution

From 24 to 12943 there are

$$12943 - 24 + 1 = 12920 \text{ numbers.}$$

From 24 to 12943 only the digit 1 can repeat five times, hence there is one number 11111.

From 24 to 12943 each of digits 1, 3, 5 can repeat four times and numbers 1111, 3333, 5555, 12111, 10111 satisfy this condition, then five numbers in total.

From 24 to 12943 among three-digit numbers integers 111, 333, 555 can appear three times, then three numbers in total.

From 24 to 12943 among four-digit numbers there are 27+27+27=81 integers which appear three times, namely

1011 1101 1110 1121 1131 1141 1151 1161 1171 1181 1191 1211 1311

1411 1511 1611 1711 1811 1911 1112 1113 1114 1115 1116 1117 1118 1119

3033 3303 3330 3323 3313 3343 3353 3363 3373 3383 3393 3233 3133

3433 3533 3633 3733 3833 3933 3332 3331 3334 3335 3336 3337 3338 3339

5055 5505 5550 5525 5535 5545 5515 5565 5575 5585 5595 5255 5355

5455 5155 5655 5755 5855 5955 5552 5551 5554 5553 5556 5557 5558 5559

From 24 to 12943 among five-digit numbers consider several cases.

Let the digit 1 in the number 10*abc* appears three times (1 occupies two places among *a, b, c,* the third place may contain any digit out of 0, 2, 3, 4, 5, 6, 7, 8, 9), there are 28 numbers satisfying this condition, namely

10011 10211 10311 10411 10511 10611 10711 10811 10911

10101 10110 10111 10112 10113 10114 10115 10116 10117 10118 10119

10121 10131 10141 10151 10161 10171 10181 10191

Let the digit 1 in the number 11*abc* appears three times, there are 243 numbers satisfying this condition. To prove it, one has to consider two cases, namely,

1) the case where two positions excluding the digit 1 coincide (numbers 11*aa*1, 11*a*1*a*, 111*aa*), there are 3*9=27 numbers satisfying this condition;
2) the case where two positions excluding the digit 1 are different (numbers 11*ab*1, 111*ab*, 11*a*1*b*), there are 3*9*8=216 numbers satisfying this condition.

In conclusion, 27+216=243 numbers satisfy cases 1) or 2).

Let the digit 1 in the number 12*abc* appears three times (1 occupies two places among *a, b, c,* the third place may contain any digit out of 0, 2, 3, 4, 5, 6, 7, 8, 9), there are 28 numbers satisfying this condition, namely

12011 12211 12311 12411 12511 12611 12711 12811 12911

12101 12110 12111 12112 12113 12114 12115 12116 12117 12118 12119

12121 12131 12141 12151 12161 12171 12181 12191

Therefore, there are

$$1 + 5 + 3 + 27 + 27 + 27 + 28 + 27 + 216 + 28 = 389$$

numbers from 24 to 12943 such that 1, 3, 5 appear 3 or more than 3 times.

Therefore, there are

$$12920 - 389 = 12531$$

numbers from 24 to 12943 such that 1, 3, 5 do not appear 3 or more than 3 times.

Answer: 12531 numbers.

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