Answer to Question #89431 – Math – Differential Equations

Question

Solve the second order differential equation d^2y/dx^2-4y=12x, y(0)=4, y'(0)=1

Solution

Let y=p + q where p and q are the function of x, then y”=p” + q”

So, p”+q”-4p-4q=12x

If p”-4p=0 can be solved we only have to solve q”-4q=12x

p”-4p=0 has the characteristics equation (m-2)(m+2)=0 and we use the roots to suggest what p might be:

p=Ae^(2x)+Be^(-2x);

p’=2Ae^(2x)-2Be^(-2x)

p”=4Ae^(2x)+4Be^(-2x)=4p.

Now, for q. Since the right-hand side of the differential equation is 12x, q= ax+ b and q”=0, q’=a, a=constant;

q= ax+ b

q’=a

q”=0 (Derivative of the constant is equal to zero)

q”- 4q must be equal to 12x so -4ax-4b=12x, making b=0 and a=-3

Therefore,

$q=-3x$

$y = p + q = Ae^{(2x)}+Be^{(-2x)}-3x$

$y' = 2Ae^{(2x)}-2Be^{(-2x)}-3$

After putting values,

$y(0)=4=A+B$

$y'(0)=1=2A-2B-3$

So, we have

$A=3$ and $B=1$

$Y=3e^{(2x)}+e^{(-2x)}-3x$

Answer: $Y=3e^{(2x)}+e^{(-2x)}-3x$.