Answer to Question #89430 – Math – Differential Equations

Question

solve dy/dx=-1-x^2+y^2

Solution

$$\frac{dy}{dx}=-1-x^2+y^2$$

As the above equation is in the form of the Ricotta Equation, So Its particular solution is given as:

$$\frac{dy}{dx}=A(x)\ y2+B(x)\ y+\ C(x)$$

$$Y=y_1+\frac{1}{v}$$

$$\frac{dy}{dx}=\frac{dy_1}{dx}-\frac{1}{v^2}\frac{dv}{dx}$$

$$\frac{dy}{dx}=y^2-(1+x^2)$$

Here,

A(x) =1 B(x)=0 C(x)=-(x^2+1)

Y1=-x is the particular solution of the differential equation because it is verifying the differential equation

For checking y=-x

So y'=-1

Y'=y^2-(x^2+1)

-1=(-x)2-(x^2+1)

-1=-1 checked and verified

Y=y'+$\frac{1}{v}$

y=-x+$\frac{1}{v}$

$$\frac{dy}{dx} = -1 - \frac{1}{v^2}\frac{dv}{dx}$$

$$\frac{dy}{dx} = y^2 - (x^2 + 1)$$

-1-$\frac{1}{v^2}\frac{dv}{dx} = \left(\frac{1}{v} - x^2\right)2 - x^2$-1

-1-$\frac{1}{v^2}\frac{dv}{dx}$=$\frac{1}{v^2} - \frac{2x}{v} + x^2 - x^2 + 1$

After Simplifying we have,

$$\frac{dv}{dx} = -1 + 2xv$$

$\frac{dv}{dx} + v(-2x) = -1$ .... Equation (1)

It is the form of $\frac{dv}{dx} + vP(x) = Q(x)$

Integrating Factor =$e^{\int P(x)dx}$

=$e^{\int -2xdx}$

=e^{-x2}

Multiplying Integration factor by equation(1)

$$e^{-x2}(\frac{dv}{dx} - 2xv) = -e^{-x2}$$

$\frac{d}{dx}(v.e^{-x2}) = -e^{-x2}(\frac{d}{dx}(V.I.F)$=f(x)

$$\int \frac{d}{dx}(v.e^{-x2}) = \int -e^{-x2}\, dx$$

$(v.e^{-x2})$ =-$\frac{1}{2}\sqrt{\pi}\, Erf(x) \quad + c$

V=$\frac{1}{x+y}$

$\frac{1}{x+y}.\, e^{-x^2}$ = -$\frac{1}{2}\sqrt{\pi}\, Erf(x) \quad + c$

So, we have

$$y = \frac{e^{-x2}}{c-\frac{1}{2}\sqrt{\pi}\, Erf(x)} - x$$